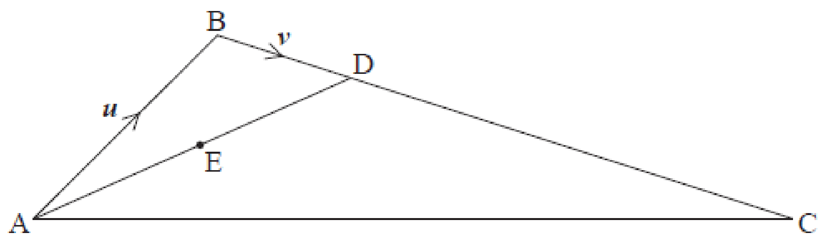
SL Paper 1

In the following diagram, $\boldsymbol{u} = \overrightarrow{\mathrm{AB}}$ and $\boldsymbol{v} = \overrightarrow{\mathrm{BD}}$.



The midpoint of $\overrightarrow{\mathrm{AD}}$ is E and $\frac{\mathrm{BD}}{\mathrm{DC}} = \frac{1}{3}$.

Express each of the following vectors in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$.

a. $\overrightarrow{\mathrm{AE}}$ [3]

b. $\overrightarrow{\mathrm{EC}}$ [4]

Markscheme

a. $\overrightarrow{\mathrm{AE}} = \frac{1}{2}\overrightarrow{\mathrm{AD}}$ ***A1***

attempt to find $\overrightarrow{\mathrm{AD}}$ ***M1***

e.g. $\overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BD}}$, $\boldsymbol{u} + \boldsymbol{v}$

$\overrightarrow{\mathrm{AE}} = \frac{1}{2}(u + v) \left(= \frac{1}{2}\boldsymbol{u} + \frac{1}{2}\boldsymbol{v}\right)$ ***A1 N2***

[3 marks]

b. $\overrightarrow{\mathrm{EC}} = \overrightarrow{\mathrm{AE}} = \frac{1}{2}(\boldsymbol{u} + \boldsymbol{v})$ ***A1***

$\overrightarrow{\mathrm{DC}} = 3\boldsymbol{v}$ ***A1***

attempt to find $\overrightarrow{\mathrm{EC}}$ ***M1***

e.g. $\overrightarrow{\mathrm{ED}} + \overrightarrow{\mathrm{DC}}$, $\frac{1}{2}(\boldsymbol{u} + \boldsymbol{v}) + 3\boldsymbol{v}$

$\overrightarrow{\mathrm{EC}} = \frac{1}{2}\boldsymbol{u} + \frac{7}{2}\boldsymbol{v} \left(= \frac{1}{2}(\boldsymbol{u} + 7\boldsymbol{v})\right)$ ***A1 N2***

[4 marks]

Examiners report

a. [N/A]

b. [N/A]

Let $\boldsymbol{u} = -3\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{v} = m\boldsymbol{j} + n\boldsymbol{k}$, where $m,\ n \in \mathbb{R}$. Given that $\boldsymbol{v}$ is a unit vector perpendicular to $\boldsymbol{u}$, find the possible values of m and of n.

Markscheme

correct scalar product ***(A1)***

eg $m + n$

setting up their scalar product equal to 0 (seen anywhere) ***(M1)***

eg $\boldsymbol{u} \bullet \boldsymbol{v} = 0,\ -3(0) + 1(m) + 1(n) = 0,\ m = -n$

correct interpretation of unit vector ***(A1)***

eg $\sqrt{0^2 + m^2 + n^2} = 1,\ m^2 + n^2 = 1$

valid attempt to solve their equations (must be in one variable) ***M1***

eg $(-n)^2 + n^2 = 1,\ \sqrt{1 - n^2} + n = 0,\ m^2 + (-m)^2 = 1,\ m - \sqrt{1 - m^2} = 0$

correct working ***A1***

eg $2n^2 = 1,\ 2m^2 = 1,\ \sqrt{2} = \frac{1}{n},\ m = \pm\frac{1}{\sqrt{2}}$

both correct pairs ***A2*** ***N3***

eg $m = \frac{1}{\sqrt{2}}$ and $n = -\frac{1}{\sqrt{2}},\ m = -\frac{1}{\sqrt{2}}$ and $n = \frac{1}{\sqrt{2}},$

$m = (0.5)^{\frac{1}{2}}$ and $n = -(0.5)^{\frac{1}{2}},\ m = -\sqrt{\frac{1}{2}}$ and $n = \sqrt{\frac{1}{2}}$

Note: Award ***A0*** for $m = \pm\frac{1}{\sqrt{2}},\ n = \pm\frac{1}{\sqrt{2}}$, or any other answer that does not clearly indicate the correct pairs.

[7 marks]

Examiners report

Most of the candidates recognized that the scalar product of the vectors must be zero. However, some did not find the correct scalar product because they did not multiply the correct corresponding vector components of $\boldsymbol{u}$ and $\boldsymbol{v}$. In addition, the majority of candidates did not attempt to use the fact that the unit vector $\boldsymbol{v}$ has a magnitude of 1. For the small number of candidates who were successful in solving for m and/or n, some did not clearly present the correct pairs of answers.

The vectors $\boldsymbol{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} k+3 \\ k \end{pmatrix}$ are perpendicular to each other.

a. Find the value of k. [4]

b. Given that $\boldsymbol{c} = \boldsymbol{a} + 2\boldsymbol{b}$, find $\boldsymbol{c}$. [3]

Markscheme

a. evidence of scalar product ***M1***

eg $\boldsymbol{a} \bullet \boldsymbol{b}$, $4(k+3)+2k$

recognizing scalar product must be zero ***(M1)***

eg $\boldsymbol{a} \bullet \boldsymbol{b} = 0,\ 4k+12+2k=0$

correct working (must involve combining terms) ***(A1)***

eg $6k+12,\ \ 6k=-12$

$k=-2$ ***A1*** ***N2***

[4 marks]

b. attempt to substitute **their** value of k (seen anywhere) ***(M1)***

eg $\boldsymbol{b} = \begin{pmatrix} -2+3 \\ -2 \end{pmatrix}$, $2\boldsymbol{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

correct working ***(A1)***

eg $\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}$, $\begin{pmatrix} 4+2k+6 \\ 2+2k \end{pmatrix}$

$\boldsymbol{c} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$ ***A1*** ***N2***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

Note: In this question, distance is in metres and time is in seconds.

Two particles P_1 and P_2 start moving from a point A at the same time, along different straight lines.

After t seconds, the position of P_1 is given by $\boldsymbol{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

Two seconds after leaving A, P_1 is at point B.

Two seconds after leaving A, P_2 is at point C, where $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$.

a. Find the coordinates of A. [2]

b.i. Find $\overrightarrow{\mathrm{AB}}$; [3]

b.ii. Find $\left|\overrightarrow{\mathrm{AB}}\right|$. [2]

c. Find $\cos \mathrm{B\hat{A}C}$. [5]

d. Hence or otherwise, find the distance between P_1 and P_2 two seconds after they leave A. [4]

Markscheme

a. recognizing $t = 0$ at A ***(M1)***

A is $(4,\ -1,\ 3)$ ***A1 N2***

[2 marks]

b.i. **METHOD 1**

valid approach ***(M1)***

eg $\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix},\ (6,\ 3,\ -1)$

correct approach to find $\overrightarrow{\mathrm{AB}}$ ***(A1)***

eg $\mathrm{AO} + \mathrm{OB},\ \mathrm{B} - \mathrm{A},\ \begin{pmatrix} 6 \\ 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$ ***A1 N2***

METHOD 2

recognizing $\overrightarrow{\mathrm{AB}}$ is two times the direction vector ***(M1)***

correct working ***(A1)***

eg $\overrightarrow{\mathrm{AB}} = 2\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$ ***A1 N2***

[3 marks]

b.ii. correct substitution ***(A1)***

eg $\left|\overrightarrow{\mathrm{AB}}\right| = \sqrt{2^2 + 4^2 + 4^2},\ \sqrt{4 + 16 + 16},\ \sqrt{36}$

$\left|\overrightarrow{\mathrm{AB}}\right| = 6$ ***A1 N2***

[2 marks]

c. **METHOD 1 (vector approach)**

valid approach involving $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$ ***(M1)***

eg $\overrightarrow{\mathrm{AB}} \bullet \overrightarrow{\mathrm{AC}},\ \frac{\overrightarrow{\mathrm{BA}} \bullet \overrightarrow{\mathrm{AC}}}{\mathrm{AB} \times \mathrm{AC}}$

finding scalar product and $\left|\overrightarrow{\mathrm{AC}}\right|$ ***(A1)(A1)*** scalar product $2(3)+4(0)-4(4)\ (=-10)$

$\left|\overrightarrow{\mathrm{AC}}\right|=\sqrt{3^2+0^2+4^2}\ (=5)$

substitution of **their** scalar product and magnitudes into cosine formula ***(M1)***

eg $\cos \mathrm{B\hat{A}C}=\frac{6+0-16}{6\sqrt{3^2+4^2}}$

$\cos B\hat{A}C=-\frac{10}{30}\left(=-\frac{1}{3}\right)$ ***A1 N2***

METHOD 2 (triangle approach)

valid approach involving cosine rule ***(M1)***

eg $\cos \mathrm{B\hat{A}C}=\frac{\mathrm{AB}^2+\mathrm{AC}^2-\mathrm{BC}^2}{2\times \mathrm{AB}\times \mathrm{AC}}$

finding lengths AC and BC ***(A1)(A1)***

$\mathrm{AC}=5,\ \mathrm{BC}=9$

substitution of **their** lengths into cosine formula ***(M1)***

eg $\cos \mathrm{B\hat{A}C}=\frac{5^2+6^2-9^2}{2\times 5\times 6}$

$\cos \mathrm{B\hat{A}C}=-\frac{20}{60}\ \left(=-\frac{1}{3}\right)$ ***A1 N2***

[5 marks]

d. **Note:** Award relevant marks for working seen to find BC in part (c) (if cosine rule used in part (c)).

METHOD 1 (using cosine rule)

recognizing need to find BC ***(M1)***

choosing cosine rule ***(M1)***

eg $c^2=a^2+b^2-2ab\cos \mathrm{C}$

correct substitution into RHS ***A1***

eg $\mathrm{BC}^2=(6)^2+(5)^2-2(6)(5)\left(-\frac{1}{3}\right),\ 36+25+20$

distance is 9 ***A1 N2***

METHOD 2 (finding magnitude of $\overrightarrow{BC}$)

recognizing need to find BC ***(M1)***

valid approach ***(M1)***

eg attempt to find $\overrightarrow{\mathrm{OB}}$ or $\overrightarrow{\mathrm{OC}}$, $\overrightarrow{\mathrm{OB}}=\begin{pmatrix}6\\3\\-1\end{pmatrix}$ or $\overrightarrow{\mathrm{OC}}=\begin{pmatrix}7\\-1\\7\end{pmatrix}$, $\overrightarrow{\mathrm{BA}}+\overrightarrow{\mathrm{AC}}$

correct working ***A1***

eg $\overrightarrow{\mathrm{BC}}=\begin{pmatrix}1\\-4\\8\end{pmatrix},\ \overrightarrow{\mathrm{CB}}=\begin{pmatrix}-1\\4\\-8\end{pmatrix},\ \sqrt{1^2+4^2+8^2}=\sqrt{81}$

distance is 9 ***A1 N2***

METHOD 3 (finding coordinates and using distance formula)

recognizing need to find BC ***(M1)***

valid approach ***(M1)***

eg attempt to find coordinates of B or C, $\mathrm{B}(6,\ 3,\ -1)$ or $\mathrm{C}(7,\ -1,\ 7)$

correct substitution into distance formula ***A1***

eg $\mathrm{BC} = \sqrt{(6-7)^2 + (3-(-1))^2 + (-1-7)^2},\ \sqrt{1^2+4^2+8^2} = \sqrt{81}$

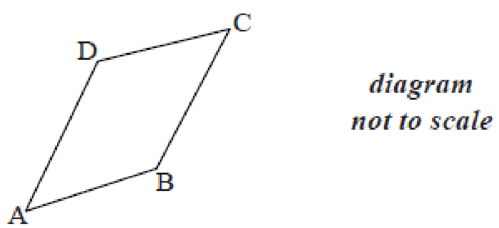
distance is 9 ***A1*** ***N2***

[4 marks]

Examiners report

a. [N/A]
b.i. [N/A]
b.ii. [N/A]
c. [N/A]
d. [N/A]

The following diagram shows quadrilateral ABCD, with $\overrightarrow{\mathrm{AD}} = \overrightarrow{\mathrm{BC}}$, $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, and $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$.



a. Find $\overrightarrow{\mathrm{BC}}$. [2]

b. Show that $\overrightarrow{\mathrm{BD}} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$. [2]

c. Show that vectors $\overrightarrow{\mathrm{BD}}$ and $\overrightarrow{\mathrm{AC}}$ are perpendicular. [3]

Markscheme

a. evidence of appropriate approach ***(M1)***

e.g. $\overrightarrow{\mathrm{AC}} - \overrightarrow{\mathrm{AB}}$, $\begin{pmatrix} 4-3 \\ 4-1 \end{pmatrix}$

$\overrightarrow{\mathrm{BC}} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ ***A1*** ***N2***

[2 marks]

b. **METHOD 1**

$\overrightarrow{\mathrm{AD}} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ ***(A1)*** correct approach ***A1***

e.g. $\overrightarrow{\mathrm{AD}} - \overrightarrow{\mathrm{AB}}$, $\begin{pmatrix} 1-3 \\ 3-1 \end{pmatrix}$

$\overrightarrow{\mathrm{BD}} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ ***AG N0***

METHOD 2

recognizing $\overrightarrow{\mathrm{CD}} = \overrightarrow{\mathrm{BA}}$ ***(A1)***

correct approach ***A1***

e.g. $\overrightarrow{\mathrm{BC}} + \overrightarrow{\mathrm{CD}}$, $\begin{pmatrix} 1-3 \\ 3-1 \end{pmatrix}$

$\overrightarrow{\mathrm{BD}} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ ***AG N0***

[2 marks]

c. **METHOD 1**

evidence of scalar product ***(M1)***

e.g. $\overrightarrow{\mathrm{BD}} \bullet \overrightarrow{\mathrm{AC}}$, $\begin{pmatrix} -2 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

correct substitution ***A1***

e.g. $(-2)(4) + (2)(4)$, $-8 + 8$

$\overrightarrow{\mathrm{BD}} \bullet \overrightarrow{\mathrm{AC}} = 0$ ***A1***

therefore vectors $\overrightarrow{\mathrm{BD}}$ and $\overrightarrow{\mathrm{AC}}$ are perpendicular ***AG N0***

METHOD 2

attempt to find angle between two vectors ***(M1)***

e.g. $\frac{\boldsymbol{a} \bullet \boldsymbol{b}}{ab}$

correct substitution ***A1***

e.g. $\frac{(-2)(4)+(2)(4)}{\sqrt{8}\sqrt{32}}$, $\cos\theta = 0$

$\theta = 90^\circ$ ***A1***

therefore vectors $\overrightarrow{\mathrm{BD}}$ and $\overrightarrow{\mathrm{AC}}$ are perpendicular ***AG N0***

[3 marks]

Examiners report

a. This question on two-dimensional vectors was generally very well done.

b. This question on two-dimensional vectors was generally very well done. A very small number of candidates had trouble with the "show that" in part (b) of the question.

c. Nearly all candidates knew to use the scalar product in part (c) to show that the vectors are perpendicular.

Let $\vec{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\vec{AB} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$, where O is the origin. L_1 is the line that passes through A and B.

a. Find a vector equation for L_1. [2]

b. The vector $\begin{pmatrix} 2 \\ p \\ 0 \end{pmatrix}$ is perpendicular to $\vec{AB}$. Find the value of *p*. [3]

Markscheme

a. any correct equation in the form ***r*** = ***a*** + *t**b*** (accept any parameter for *t*)

where ***a*** is $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, and ***b*** is a scalar multiple of $\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$ ***A2 N2***

eg ***r*** = $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = t \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$, ***r*** = 2***i*** + ***j*** + 3***k*** + *s*(***i*** + 3***j*** + ***k***)

Note: Award ***A1*** for the form ***a*** + *t**b***, ***A1*** for the form *L* = ***a*** + *t**b***, A0 for the form ***r*** = ***b*** + *t**a***.

[2 marks]

b. **METHOD 1**

correct scalar product ***(A1)***

eg (1 × 2) + (3 × *p*) + (1 × 0), 2 + 3*p*

evidence of equating **their** scalar product to zero ***(M1)***

eg ***a*•*b*** = 0, 2 + 3p = 0, 3*p* = −2

$p = -\frac{2}{3}$ ***A1 N3***

METHOD 2

valid attempt to find angle between vectors ***(M1)***

correct substitution into numerator and/or angle ***(A1)***

eg $\cos\theta = \frac{(1\times2)+(3\times p)+(1\times0)}{|a||b|}, \ \cos\theta = 0$

$p = -\frac{2}{3}$ ***A1 N3***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

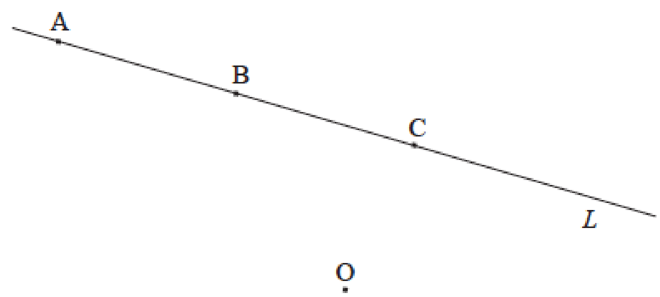
A line L passes through points $A(-2,\ 4,\ 3)$ and $B(-1,\ 3,\ 1)$.

a. (i) Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$. [3]

(ii) Find $\left|\overrightarrow{\mathrm{AB}}\right|$.

b. Find a vector equation for L. [2]

c. The following diagram shows the line L and the origin O. The point C also lies on L. [4]



Point C has position vector $\begin{pmatrix} 0 \\ y \\ -1 \end{pmatrix}$.

Show that $y = 2$.

d. (i) Find $\overrightarrow{\mathrm{OC}} \bullet \overrightarrow{\mathrm{AB}}$. [3]

(ii) Hence, write down the size of the angle between C and L.

e. Hence or otherwise, find the area of triangle OAB. [4]

Markscheme

a. (i) correct approach ***A1***

eg $\mathrm{B} - \mathrm{A}, \mathrm{AO} + \mathrm{OB}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ ***AG N0***

(ii) correct substitution ***(A1)***

eg $\sqrt{(1)^2 + (-1)^2 + (-2)^2}, \sqrt{1+1+4}$

$\left|\overrightarrow{\mathrm{AB}}\right| = \sqrt{6}$ ***A1 N2***

[3 marks]

b. any correct equation in the form $r = a + tb$ (any parameter for t)

where a is $\begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$ or $\begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$ and b is a scalar multiple of $\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ ***A2 N2***

eg \(r = \left({

-2
4
3

} \right) + t\left({

$$\begin{matrix} 1 \\ -1 \\ -2 \end{matrix}$$

} \right),{\text{ }}(x,{\text{ }}y,{\text{ }}z) = (- 1,{\text{ }}3,{\text{ }}1) + t(1,{\text{ }} - 1,{\text{ }} - 2),{\text{ }}{\mathbf{r}} = \left({

$$\begin{matrix} -1+t \\ 3-t \\ 1-2t \end{matrix}$$

} \right)\)

Note: Award ***A1*** for the form $\mathbf{a}+t\mathbf{b}$, ***A1*** for the form $L=\mathbf{a}+t\mathbf{b}$, ***A0*** for the form $\mathbf{r}=\mathbf{b}+t\mathbf{a}$.

c. **METHOD 1**

valid approach ***(M1)***

eg $\begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ y \\ -1 \end{pmatrix},\ \begin{pmatrix} 0 \\ y \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} + s\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$

one correct equation from **their** approach ***A1***

eg $-1+t=0,\ 1-2t=-1,\ -2+s=0,\ 3-2s=-1$

one correct value for **their** parameter and equation ***A1***

eg $t=1,\ s=2$

correct substitution ***A1***

eg $3+1(-1),\ 4+2(-1)$

$y=2$ ***AG*** ***N0***

METHOD 2

valid approach ***(M1)***

eg $\overrightarrow{\mathrm{AC}}=k\overrightarrow{\mathrm{AB}}$

correct working ***A1***

eg $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 2 \\ y-4 \\ -4 \end{pmatrix},\ \begin{pmatrix} 2 \\ y-4 \\ -4 \end{pmatrix} = k\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$

$k=2$ ***A1***

correct substitution ***A1***

eg $y-4=-2$

$y=2$ ***AG*** ***N0***

[4 marks]

d. (i) correct substitution ***A1***

eg $0(1)+2(-1)-1(-2),\ 0-2+2$

$\overrightarrow{\mathrm{OC}} \bullet \overrightarrow{\mathrm{AB}} = 0$ ***A1*** ***N1***

(ii) 90° or $\frac{\pi}{2}$ ***A1*** ***N1***

[3 marks]

e. **METHOD 1** $(\text{area} = 0.5 \times \text{height} \times \text{base})$

$\left|\overrightarrow{\mathrm{OC}}\right| = \sqrt{0 + 2^2 + (-1)^2} \quad \left(= \sqrt{5}\right)$ (seen anywhere) ***A1***

valid approach ***(M1)***

eg $\frac{1}{2} \times \left|\overrightarrow{\mathrm{AB}}\right| \times \left|\overrightarrow{\mathrm{OC}}\right|, \ \left|\overrightarrow{\mathrm{OC}}\right|$ is height of triangle

correct substitution ***A1***

eg $\frac{1}{2} \times \sqrt{6} \times \sqrt{0 + (2)^2 + (-1)^2}, \ \frac{1}{2} \times \sqrt{6} \times \sqrt{5}$

area is $\frac{\sqrt{30}}{2}$ ***A1 N2***

METHOD 2 (difference of two areas)

one correct magnitude (seen anywhere) ***A1***

eg $\left|\overrightarrow{\mathrm{OC}}\right| = \sqrt{2^2 + (-1)^2} \quad \left(= \sqrt{5}\right), \quad \left|\overrightarrow{\mathrm{AC}}\right| = \sqrt{4 + 4 + 16} \quad \left(= \sqrt{24}\right), \quad \left|\overrightarrow{\mathrm{BC}}\right| = \sqrt{6}$

valid approach ***(M1)***

eg $\Delta\mathrm{OAC} - \Delta\mathrm{OBC}$

correct substitution ***A1***

eg $\frac{1}{2} \times \sqrt{24} \times \sqrt{5} - \frac{1}{2} \times \sqrt{5} \times \sqrt{6}$

area is $\frac{\sqrt{30}}{2}$ ***A1 N2***

METHOD 3 $(\text{area} = \frac{1}{2}ab\sin C \text{ for } \Delta\mathrm{OAB})$

one correct magnitude of $\overrightarrow{\mathrm{OA}}$ or $\overrightarrow{\mathrm{OB}}$ (seen anywhere) ***A1***

eg $\left|\overrightarrow{\mathrm{OA}}\right| = \sqrt{(-2)^2 + 4^2 + 3^2} \quad \left(= \sqrt{29}\right), \quad \left|\overrightarrow{\mathrm{OB}}\right| = \sqrt{1 + 9 + 1} \quad \left(= \sqrt{11}\right)$

valid attempt to find $\cos\theta$ ***or*** $\sin\theta$ ***(M1)***

eg $\cos \mathrm{C} = \frac{-1-3-2}{\sqrt{6}\times\sqrt{11}} \quad \left(= \frac{-6}{\sqrt{66}}\right), \quad 29 = 6 + 11 - 2\sqrt{6}\sqrt{11}\cos\theta, \ \frac{\sin\theta}{\sqrt{5}} = \frac{\sin 90}{\sqrt{29}}$

correct substitution into $\frac{1}{2}ab\sin \mathrm{C}$ ***A1***

eg $\frac{1}{2} \times \sqrt{6} \times \sqrt{11} \times \sqrt{1 - \frac{36}{66}}, \ 0.5 \times \sqrt{6} \times \sqrt{29} \times \frac{\sqrt{5}}{\sqrt{29}}$

area is $\frac{\sqrt{30}}{2}$ ***A1 N2***

[4 marks]

Total [16 marks]

Examiners report

a. Finding $\overrightarrow{\mathrm{AB}}$ and its magnitude were mostly well done.

b. Mostly correct answers with common errors being using both position vectors or writing it as "$L =$" instead of "$\mathbf{r} =$".

c. Many candidates assumed that $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{BC}}$, although this was not indicated on the diagram nor given in the question.

d. Mostly this was well answered. A surprising number of candidates wrote the scalar product as a vector $(0, \ -2, \ 2)$. In part b) many missed the clue given by the phrase "hence, write down" and carried out a calculation for cosine theta using the scalar product again.

e. This part was poorly done. Few candidates realised how to directly calculate the area based on their previous work and could not see the "height" of the obtuse triangle as $\left|\overrightarrow{\mathrm{OC}}\right|$. Those who tried to use $A = \frac{1}{2}ab\sin C$ had trouble generating the angle. Those who subtracted areas $(\Delta\mathrm{OAC} - \Delta\mathrm{OBC})$ were usually successful.

Let $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$.

a. Find $\overrightarrow{\mathrm{BC}}$. [2]

b. Find a unit vector in the direction of $\overrightarrow{\mathrm{AB}}$. [3]

c. Show that $\overrightarrow{\mathrm{AB}}$ is perpendicular to $\overrightarrow{\mathrm{AC}}$. [3]

Markscheme

a. evidence of appropriate approach ***(M1)***

e.g. $\overrightarrow{\mathrm{BC}} = \overrightarrow{\mathrm{BA}} + \overrightarrow{\mathrm{AC}}$, $\begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix} - \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$

$\overrightarrow{\mathrm{BC}} = \begin{pmatrix} -8 \\ -1 \\ -1 \end{pmatrix}$ ***A1 N2***

[2 marks]

b. attempt to find the length of $\overrightarrow{\mathrm{AB}}$ ***(M1)***

$|\overrightarrow{\mathrm{AB}}| = \sqrt{6^2 + (-2)^2 + 3^2} \, (= \sqrt{36+4+9} = \sqrt{49} = 7)$ ***(A1)***

unit vector is $\frac{1}{7}\begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} \left(= \begin{pmatrix} \frac{6}{7} \\ -\frac{2}{7} \\ \frac{3}{7} \end{pmatrix} \right)$ ***A1 N2***

[3 marks]

c. recognizing that the dot product or $\cos\theta$ being 0 implies perpendicular ***(M1)***

correct substitution in a scalar product formula ***A1***

e.g. $(6) \times (-2) + (-2) \times (-3) + (3) \times (2)$, $\cos\theta = \frac{-12+6+6}{7\times\sqrt{17}}$

correct calculation ***A1***

e.g. $\overrightarrow{\mathrm{AB}} \bullet \overrightarrow{\mathrm{AC}} = 0$, $\cos\theta = 0$

therefore, they are perpendicular ***AG N0***

[3 marks]

Examiners report

a. Part (a) was generally done well with candidates employing different correct methods to find the vector $\overrightarrow{\mathrm{BC}}$. Some candidates subtracted the given vectors in the wrong order and others simply added them. Calculation errors were seen with some frequency.

b. Many candidates did not appear to know how to find a unit vector in part (b). Some tried to write down the vector equation of a line, indicating no familiarity with the concept of unit vectors while others gave the vector (1, 1, 1) or wrote the same vector $\overrightarrow{\mathrm{AB}}$ as a linear combination of $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$. A number of candidates correctly found the magnitude but did not continue on to write the unit vector.

c. Candidates were generally successful in showing that the vectors in part (c) were perpendicular. Many used the efficient approach of showing that the scalar product equalled zero, while others worked a little harder than necessary and used the cosine rule to find the angle between the two vectors.

Consider points $\mathrm{A}(1, -2, -1)$, $\mathrm{B}(7, -4, 3)$ and $\mathrm{C}(1, -2, 3)$. The line L_1 passes through C and is parallel to $\overrightarrow{\mathrm{AB}}$.

A second line, L_2 , is given by $\boldsymbol{r} = \begin{pmatrix} -1 \\ 2 \\ 15 \end{pmatrix} + s\begin{pmatrix} 3 \\ -3 \\ p \end{pmatrix}$.

a.i. Find $\overrightarrow{\mathrm{AB}}$. [2]

a.ii. Hence, write down a vector equation for L_1 . [2]

b. Given that L_1 is perpendicular to L_2 , show that $p = -6$. [3]

c. The line L_1 intersects the line L_2 at point Q. Find the x-coordinate of Q. [7]

Markscheme

a.i. valid approach ***(M1)***

eg $\begin{pmatrix} 7 \\ -4 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, $\mathrm{A} - \mathrm{B}$, $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{AO}} + \overrightarrow{\mathrm{OB}}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$ ***A1 N2***

[2 marks]

a.ii. any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ and $\boldsymbol{b}$ is a scalar multiple of $\overrightarrow{\mathrm{AB}}$ ***A2 N2*** *eg* $\boldsymbol{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$, $(x, y, z) = (1, -2, 3) + t(3, -1, 2)$, $\boldsymbol{r} = \begin{pmatrix} 1+6t \\ -2-2t \\ 3+4t \end{pmatrix}$

Note: Award ***A1*** for $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** for $L_1 = \boldsymbol{a} + t\boldsymbol{b}$, ***A0*** for $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[2 marks]

b. recognizing that scalar product $= 0$ (seen anywhere) ***R1***

correct calculation of scalar product ***(A1)***

eg $6(3) - 2(-3) + 4p$, $18 + 6 + 4p$

correct working ***A1***

eg $24 + 4p = 0$, $4p = -24$

$p = -6$ ***AG*** ***N0***

[3 marks]

c. setting lines equal ***(M1)***

eg $L_1 = L_2$, $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 15 \end{pmatrix} + s\begin{pmatrix} 3 \\ -3 \\ -6 \end{pmatrix}$

any two correct equations with **different** parameters ***A1A1***

eg $1 + 6t = 1 + 3s$, $-2 - 2t = 2 - 3s$, $3 + 4t = 15 - 6s$

attempt to solve **their** simultaneous equations ***(M1)***

one correct parameter ***A1***

eg $t = \frac{1}{2}$, $s = \frac{5}{3}$

attempt to substitute parameter into vector equation ***(M1)***

eg $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$, $1 + \frac{1}{2} \times 6$

$x = 4$ (accept (4, -3, 5), ignore incorrect values for y and z) ***A1*** ***N3***

[7 marks]

Examiners report

a.i. While many candidates can find a vector given two points, few could write down a fully correct vector equation of a line.

a.ii. While many candidates can find a vector given two points, few could write down a fully correct vector equation of a line. Most candidates wrote their equation as " $L_1 =$ ", which misrepresents that the resulting equation must still be a vector.

b. Those who recognized that vector perpendicularity means the scalar product is zero found little difficulty answering part (b). Occasionally a candidate would use the given $p = 6$ to show the scalar product is zero. However, working backward from the given answer earns no marks in a question that requires candidates to show that this value is achieved.

c. While many candidates knew to set the lines equal to find an intersection point, a surprising number could not carry the process to correct completion. Some could not solve a simultaneous pair of equations, and for those who did, some did not know what to do with the parameter value. Another common error was to set the vector equations equal using the same parameter, from which the candidates did not recognize a system to solve. Furthermore, it is interesting to note that while only one parameter value is needed to answer the question, most candidates find or attempt to find both, presumably out of habit in the algorithm.

A line L_1 passes through the points $\mathrm{A}(0,\ 1,\ 8)$ and $\mathrm{B}(3,\ 5,\ 2)$.

Given that L_1 and L_2 are perpendicular, show that $p = 2$.

a.i. Find $\overrightarrow{AB}$. [2]

a.ii. Hence, write down a vector equation for L_1. [2]

b. A second line L_2, has equation ***r*** = $\begin{pmatrix} 1 \\ 13 \\ -14 \end{pmatrix} + s\begin{pmatrix} p \\ 0 \\ 1 \end{pmatrix}$. [3]

Given that L_1 and L_2 are perpendicular, show that $p = 2$.

c. The lines L_1 and L_1 intersect at $C(9,\ 13,\ z)$. Find z. [5]

d.i. Find a unit vector in the direction of L_2. [2]

d.ii Hence or otherwise, find one point on L_2 which is $\sqrt{5}$ units from C. [3]

Markscheme

a.i. valid approach ***(M1)***

eg $A - B,\ -\begin{pmatrix} 0 \\ 1 \\ 8 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \\ -6 \end{pmatrix}$ ***A1 N2***

[2 marks]

a.ii. **any** correct equation in the form ***r*** = ***a*** + *t**b*** (any parameter for t) ***A2 N2***

where ***a*** is $\begin{pmatrix} 0 \\ 1 \\ 8 \end{pmatrix}$ or $\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$, and ***b*** is a scalar multiple of $\begin{pmatrix} 3 \\ 4 \\ -6 \end{pmatrix}$

eg ***r*** = $\begin{pmatrix} 0 \\ 1 \\ 8 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ -6 \end{pmatrix}$, ***r*** = $\begin{pmatrix} 3+3t \\ 5+4t \\ 2-6t \end{pmatrix}$, ***r*** = ***j*** + 8***k*** + t(3***i*** + 4***j*** – 6***k***)

Note: Award ***A1*** for the form ***a*** + *t**b***, ***A1*** for the form ***L*** = ***a*** + *t**b***, ***A0*** for the form ***r*** = ***b*** + *t**a***.

[2 marks]

b. valid approach ***(M1)***

eg $a \bullet b = 0$

choosing correct direction vectors (may be seen in scalar product) ***A1***

eg $\begin{pmatrix} 3 \\ 4 \\ -6 \end{pmatrix}$ and $\begin{pmatrix} p \\ 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 4 \\ -6 \end{pmatrix} \bullet \begin{pmatrix} p \\ 0 \\ 1 \end{pmatrix} = 0$

correct working/equation ***A1***

eg $3p - 6 = 0$

$p = 2$ ***AG N0***

[3 marks]

c. valid approach ***(M1)***

eg $L_1 = \begin{pmatrix} 9 \\ 13 \\ z \end{pmatrix}$, $L_1 = L_2$

one correct equation (must be different parameters if both lines used) ***(A1)***

eg $3t = 9,\ 1 + 2s = 9,\ 5 + 4t = 13,\ 3t = 1 + 2s$

one correct value ***A1***

eg $t = 3,\ s = 4,\ t = 2$

valid approach to substitute their t or s value ***(M1)***

eg $8 + 3(-6),\ -14 + 4(1)$

$z = -10$ ***A1 N3***

[5 marks]

d.i. $\left|\vec{d}\right| = \sqrt{2^2 + 1}\ \left(= \sqrt{5}\right)$ ***(A1)***

$\frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \quad \left(\text{accept} \begin{pmatrix} \frac{2}{\sqrt{5}} \\ \frac{0}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix}\right)$ ***A1 N2***

[2 marks]

d.ii. **METHOD 1 (using unit vector)**

valid approach ***(M1)***

eg $\begin{pmatrix} 9 \\ 13 \\ -10 \end{pmatrix} \pm \sqrt{5}\hat{d}$

correct working ***(A1)***

eg $\begin{pmatrix} 9 \\ 13 \\ -10 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 9 \\ 13 \\ -10 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

one correct point ***A1 N2***

eg $(11,\ 13,\ -9),\ (7,\ 13,\ -11)$

METHOD 2 (distance between points)

attempt to use distance between $(1+2s,\ 13,\ -14+s)$ and $(9,\ 13,\ -10)$ ***(M1)***

eg $(2s-8)^2+0^2+(s-4)^2=5$

solving $5s^2-40s+75=0$ leading to $s=5$ or $s=3$ ***(A1)***

one correct point ***A1*** ***N2***

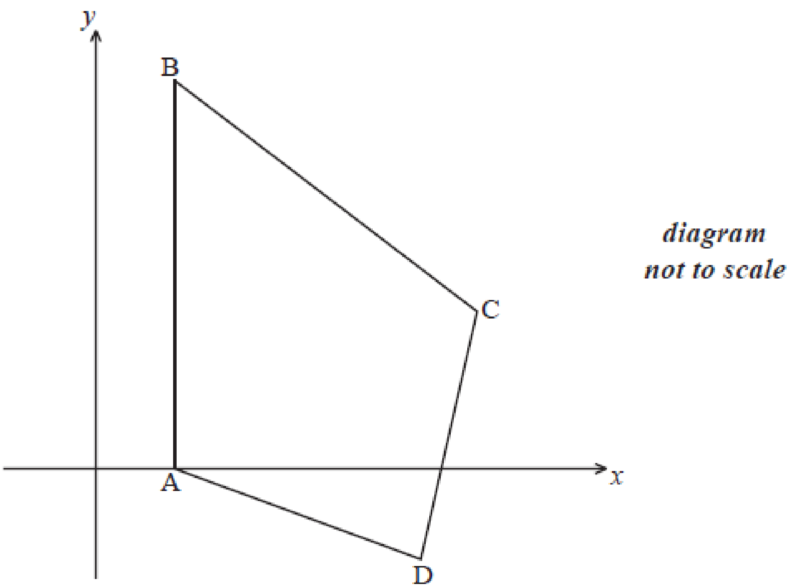
eg $(11,\ 13,\ -9),\ (7,\ 13,\ -11)$

[3 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]
c. [N/A]
d.i. [N/A]
d.ii. [N/A]

The diagram shows quadrilateral ABCD with vertices A(1, 0), B(1, 5), C(5, 2) and D(4, −1) .



a(i), (ii) and (iii). (i) Show that $\overrightarrow{\mathrm{AC}}=\begin{pmatrix}4\\2\end{pmatrix}$. [5]

(ii) Find $\overrightarrow{\mathrm{BD}}$.

(iii) Show that $\overrightarrow{\mathrm{AC}}$ is perpendicular to $\overrightarrow{\mathrm{BD}}$.

b(i) and (ii). The line (AC) has equation $\boldsymbol{r}=\boldsymbol{u}+s\boldsymbol{v}$. [4]

(i) Write down vector $\boldsymbol{u}$ and vector $\boldsymbol{v}$.

(ii) Find a vector equation for the line (BD).

c. The lines (AC) and (BD) intersect at the point $\mathrm{P}(3,\, k)$. [3]

Show that $k = 1$.

d. The lines (AC) and (BD) intersect at the point $\mathrm{P}(3,\, k)$. [5]

Hence find the area of triangle ACD.

Markscheme

a(i), (ii) and (iii)

(i) correct approach ***A1***

e.g. $\overrightarrow{\mathrm{OC}} - \overrightarrow{\mathrm{OA}}$, $\begin{pmatrix} 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ ***AG*** ***N0***

(ii) appropriate approach ***(M1)***

e.g. $\mathrm{D} - \mathrm{B}$, $\begin{pmatrix} 4 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \end{pmatrix}$, move 3 to the right and 6 down

$\overrightarrow{\mathrm{BD}} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$ ***A1*** ***N2***

(iii) finding the scalar product ***A1***

e.g. $4(3) + 2(-6)$, $12 - 12$

valid reasoning ***R1***

e.g. $4(3) + 2(-6) = 0$, scalar product is zero

$\overrightarrow{\mathrm{AC}}$ is perpendicular to $\overrightarrow{\mathrm{BD}}$ ***AG*** ***N0***

[5 marks]

b(i) and (ii)

(i) correct "position" vector for $\boldsymbol{u}$; "direction" vector for $\boldsymbol{v}$ ***A1A1*** ***N2***

e.g. $\boldsymbol{u} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$, $\boldsymbol{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $\boldsymbol{v} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$, $\boldsymbol{v} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

accept in equation e.g. $\begin{pmatrix} 5 \\ 2 \end{pmatrix} + t\begin{pmatrix} -4 \\ -2 \end{pmatrix}$

(ii) any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$, where $\boldsymbol{b} = \overrightarrow{\mathrm{BD}}$

$\boldsymbol{r} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} + t\begin{pmatrix} 3 \\ -6 \end{pmatrix}$, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + t\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ ***A2*** ***N2***

[4 marks]

c. **METHOD 1**

substitute (3, k) into equation for (AC) or (BD) ***(M1)***

e.g. $3 = 1 + 4s$, $3 = 1 + 3t$

value of t or s ***A1***

e.g. $s = \frac{1}{2}$, $-\frac{1}{2}$, $t = \frac{2}{3}$, $-\frac{1}{3}$

substituting ***A1***

e.g. $k = 0 + \frac{1}{2}(2)$

$k = 1$ ***AG*** ***N0***

METHOD 2

setting up two equations ***(M1)***

e.g. $1+4s=4+3t$, $2s=-1-6t$; setting vector equations of lines equal

value of *t* or *s* ***A1***

e.g. $s=\frac{1}{2}$, $-\frac{1}{2}$, $t=\frac{2}{3}$, $-\frac{1}{3}$

substituting ***A1***

e.g. $\boldsymbol{r}=\begin{pmatrix}4\\-1\end{pmatrix}-\frac{1}{3}\begin{pmatrix}3\\-6\end{pmatrix}$

$k=1$ ***AG*** ***N0***

[3 marks]

d. $\overrightarrow{\text{PD}}=\begin{pmatrix}1\\-2\end{pmatrix}$ ***(A1)***

$|\overrightarrow{\text{PD}}|=\sqrt{2^2+1^2}\ (=\sqrt{5})$ ***(A1)***

$|\overrightarrow{\text{AC}}|=\sqrt{4^2+2^2}\ (=\sqrt{20})$ ***(A1)***

area $=\frac{1}{2}\times|\overrightarrow{\text{AC}}|\times|\overrightarrow{\text{PD}}|\ (=\frac{1}{2}\times\sqrt{20}\times\sqrt{5})$ ***M1***

$=5$ ***A1*** ***N4***

[5 marks]

Examiners report

a(i), (ii) and (iii). The majority of candidates were successful on part (a), finding vectors between two points and using the scalar product to show two vectors to be perpendicular.

b(i) and (ii). Although a large number of candidates answered part (b) correctly, there were many who had trouble with the vector equation of a line. Most notably, there were those who confused the position vector with the direction vector, and those who wrote their equation in an incorrect form.

c. In part (c), most candidates seemed to know what was required, though there were many who made algebraic errors when solving for the parameters. A few candidates worked backward, using $k=1$, which is not allowed on a "show that" question.

d. In part (d), candidates attempted many different geometric and vector methods to find the area of the triangle. As the question said "hence", it was required that candidates should use answers from their previous working - i.e. $\text{AC}\perp\text{BD}$ and $\text{P}(3,1)$. Some geometric approaches, while leading to the correct answer, did not use "hence" or lacked the required justification.

The line L_1 passes through the points $\text{A}(2,1,4)$ and $\text{B}(1,1,5)$.

Another line L_2 has equation $\mathbf{r}=\begin{pmatrix}4\\7\\-4\end{pmatrix}+s\begin{pmatrix}0\\-1\\1\end{pmatrix}$. The lines L_1 and L_2 intersect at the point P.

a. Show that $\overrightarrow{AB} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ [1]

b(i). Hence, write down a direction vector for L_1; [1]

b(ii). Hence, write down a vector equation for L_1. [2]

c. Find the coordinates of P. [6]

d(i). Write down a direction vector for L_2. [1]

d(ii). Hence, find the angle between L_1 and L_2. [6]

Markscheme

a. correct approach ***A1***

eg $\begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$, $\mathrm{AO} + \mathrm{OB}$, $b - a$

$\overrightarrow{AB} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ ***AG N0***

[1 mark]

b(i). correct vector (or any multiple) ***A1 N1***

eg $\boldsymbol{d} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

[1 mark]

b(ii). **any** correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a}$ is $\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ ***A2 N2***

eg $\boldsymbol{r} = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} + t\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2-s \\ 1 \\ 4+s \end{pmatrix}$

Note: Award ***A1*** for $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** for $L_1 = \boldsymbol{a} + t\boldsymbol{b}$, ***A0*** for $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[2 marks]

c. valid approach ***(M1)***

eg $r_1 = r_2, \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + t\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ -4 \end{pmatrix} + s\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

one correct equation in one parameter ***A1***

eg $2 - t = 4, 1 = 7 - s, 1 - t = 4$

attempt to solve ***(M1)***

eg $2 - 4 = t, s = 7 - 1, t = 1 - 4$

one correct parameter ***A1***

eg $t = -2, s = 6, t = -3,$

attempt to substitute **their** parameter into vector equation ***(M1)*** *eg* $\begin{pmatrix}4\\7\\-4\end{pmatrix}+6\begin{pmatrix}0\\-1\\1\end{pmatrix}$

P(4, 1, 2) (accept position vector) ***A1*** ***N2***

[6 marks]

d(i). correct direction vector for L_2 ***A1*** ***N1***

eg $\begin{pmatrix}0\\-1\\1\end{pmatrix},\begin{pmatrix}0\\2\\-2\end{pmatrix}$

[1 mark]

d(ii). correct scalar product and magnitudes for **their** direction vectors ***(A1)(A1)(A1)***

scalar product $= 0\times -1 + -1\times 0 + 1\times 1\ (=1)$

magnitudes $=\sqrt{0^2+(-1)^2+1^2},\ \sqrt{-1^2+0^2+1^2}\ \left(\sqrt{2},\ \sqrt{2}\right)$

attempt to substitute **their** values into formula ***M1***

eg $\frac{0+0+1}{\left(\sqrt{0^2+(-1)^2+1^2}\right)\times\left(\sqrt{-1^2+0^2+1^2}\right)},\ \frac{1}{\sqrt{2}\times\sqrt{2}}$

correct value for cosine, $\frac{1}{2}$ ***A1***

angle is $\frac{\pi}{3}\ (=60^\circ)$ ***A1*** ***N1***

[6 marks]

Examiners report

a. [N/A]
b(i). [N/A]
b(ii). [N/A]
c. [N/A]
d(i). [N/A]
d(ii). [N/A]

Consider the points A (1 , 5 , 4) , B (3 , 1 , 2) and D (3 , k , 2) , with (AD) perpendicular to (AB) .

The point O has coordinates (0 , 0 , 0) , point A has coordinates (1 , – 2 , 3) and point B has coordinates (– 3 , 4 , 2) .

a(i) and (ii). Find [3]

(i) $\overrightarrow{\text{AB}}$;

(ii) $\overrightarrow{\text{AD}}$ giving your answer in terms of k .

[3 marks]

b. Show that $k=7$. [3]

c. The point C is such that $\overrightarrow{\text{BC}}=\frac{1}{2}\overrightarrow{\text{AD}}$. [4]

Find the position vector of C.

d. Find $\cos \mathrm{A\hat{B}C}$. [3]

Markscheme

a(i) and (ii). (i) evidence of combining vectors *(M1)*

e.g. $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}}$ (or $\overrightarrow{\mathrm{AD}} = \overrightarrow{\mathrm{AO}} + \overrightarrow{\mathrm{OD}}$ in part (ii))

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}$ ***A1 N2***

(ii) $\overrightarrow{\mathrm{AD}} = \begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix}$ ***A1 N1***

[3 marks]

b. evidence of using perpendicularity ⇒ scalar product = 0 ***(M1)***

e.g. $\begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix} = 0$

$4 - 4(k-5) + 4 = 0$ ***A1***

$-4k + 28 = 0$ (accept any correct equation clearly leading to $k = 7$) ***A1***

$k = 7$ ***AG N0***

[3 marks]

c. $\overrightarrow{\mathrm{AD}} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$ ***(A1)***

$\overrightarrow{\mathrm{BC}} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ ***A1***

evidence of correct approach ***(M1)***

e.g. $\overrightarrow{\mathrm{OC}} = \overrightarrow{\mathrm{OB}} + \overrightarrow{\mathrm{BC}}$, $\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$, $\begin{pmatrix} x-3 \\ y-1 \\ z-2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

$\overrightarrow{\mathrm{OC}} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ ***A1 N3***

[4 marks]

d. **METHOD 1**

choosing appropriate vectors, $\overrightarrow{\mathrm{BA}}$, $\overrightarrow{\mathrm{BC}}$ ***(A1)***

finding the scalar product ***M1***

e.g. $-2(1) + 4(1) + 2(-1)$, $2(1) + (-4)(1) + (-2)(-1)$

$\cos \mathrm{A\hat{B}C} = 0$ ***A1 N1***

METHOD 2

$\overrightarrow{\mathrm{BC}}$ parallel to $\overrightarrow{\mathrm{AD}}$ (may show this on a diagram with points labelled) ***R1***

$\overrightarrow{\mathrm{BC}} \perp \overrightarrow{\mathrm{AB}}$ (may show this on a diagram with points labelled) ***R1***

$\mathrm{A\hat{B}C} = 90^\circ$

$\cos \mathrm{A\hat{B}C} = 0$ ***A1*** ***N1***

[3 marks]

Examiners report

a(i) and (ii). This question was well done by many candidates. Most found $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AD}}$ correctly.

b. The majority of candidates correctly used the scalar product to show $k = 7$.

c. Some confusion arose in substituting $k = 7$ into $\overrightarrow{\mathrm{AD}}$, but otherwise part (c) was well done, though finding the position vector of C presented greater difficulty.

d. Owing to $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{BC}}$ being perpendicular, no problems were created by using these two vectors to find $\cos \mathrm{A\hat{B}C} = 0$, and the majority of candidates answering part (d) did exactly that.

A line L_1 passes through the points $\mathrm{A}(0,\ -3,\ 1)$ and $\mathrm{B}(-2,\ 5,\ 3)$.

a. (i) Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}$. [3]

(ii) Write down a vector equation for L_1.

b. A line L_2 has equation $\mathbf{r} = \begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix} + s\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$. The lines L_1 and L_2 intersect at a point C. [5]

Show that the coordinates of C are $(-1,\ 1,\ 2)$.

c. A point D lies on line L_2 so that $\left|\overrightarrow{\mathrm{CD}}\right| = \sqrt{18}$ and $\overrightarrow{\mathrm{CA}} \bullet \overrightarrow{\mathrm{CD}} = -9$. Find $\mathrm{A\hat{C}D}$. [7]

Markscheme

a. (i) correct approach ***A1***

eg $\mathrm{OB} - \mathrm{OA},\ \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix},\ \mathrm{B} - \mathrm{A}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}$ ***AG*** ***N0***

(ii) **any** correct equation in the form $r = a + tb$ (accept any parameter for t)

where a is $\begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$, and b is a scalar multiple of $\begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}$ ***A2*** ***N2***

eg$r$ = \left({

$$\begin{matrix} 0 \\ -3 \\ 1 \end{matrix}$$

} \right) + t\left({

$$\begin{matrix} -2 \\ 8 \\ 2 \end{matrix}$$

} \right),r = \left({

$$\begin{matrix} -2-2s \\ 5+8s \\ 3+2s \end{matrix}$$

} \right),$r = 2i + 5j + 3k + t(-2i + 8j + 2k)$

Note: Award ***A1*** for the form a + tb, ***A1*** for the form $L = \backslash(a$ + tb,

A0 for the form r = b + ta.

[3 marks]

b. valid approach ***(M1)***

eg equating lines, $L_1 = L_2$

one correct equation in one variable ***A1***

eg $-2t = -1, \ -2 - 2t = -1$

valid attempt to solve ***(M1)***

eg $2t = 1, \ -2t = 1$

one correct parameter ***A1***

eg $t = \frac{1}{2}, \ t = -\frac{1}{2}, \ s = -6$

correct substitution of either parameter ***A1***

eg $r = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}, \ r = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}, \ r = \begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix} - 6\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

the coordinates of C are $(-1, \ 1, \ 2)$, or position vector of C is $\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$ ***AG*** ***N0***

Note: If candidate uses the same parameter in both vector equations and working shown, award ***M1A1M1A0A0***.

[5 marks]

c. valid approach ***(M1)***

eg attempt to find $\overrightarrow{\mathrm{CA}}$, $\cos \mathrm{A\hat{C}D} = \frac{\overrightarrow{\mathrm{CA}} \bullet \overrightarrow{\mathrm{CD}}}{\left|\overrightarrow{\mathrm{CA}}\right|\left|\overrightarrow{\mathrm{CD}}\right|}$, $\mathrm{A\hat{C}D}$ formed by $\overrightarrow{\mathrm{CA}}$ and $\overrightarrow{\mathrm{CD}}$

$\overrightarrow{CA} = \begin{pmatrix} 1 \\ -4 \\ -1 \end{pmatrix}$ ***(A1)***

Notes: Exceptions to ***FT:***

1 if candidate indicates that they are finding $\overrightarrow{CA}$, but makes an error, award ***M1A0***;

2 if candidate finds an incorrect vector (including $\overrightarrow{AC}$), award ***M0A0***.

In both cases, if working shown, full ***FT*** may be awarded for subsequent correct ***FT*** work.

Award the final (***A1***) for simplification of **their** value for $A\hat{C}D$.

Award the final ***A2*** for finding **their** arc cos. If their value of cos does not allow them to find an angle, they cannot be awarded this ***A2***.

finding $\left|\overrightarrow{CA}\right|$ (may be seen in cosine formula) ***A1***

eg $\sqrt{1^2 + (-4)^2 + (-1)^2}, \sqrt{18}$

correct substitution into cosine formula ***(A1)***

eg $\frac{-9}{\sqrt{18}\sqrt{18}}$

finding $\cos A\hat{C}D - \frac{1}{2}$ ***(A1)***

$A\hat{C}D = \frac{2\pi}{3}$ (120°) ***A2*** ***N2***

Notes: Award ***A1*** if additional answers are given.

Award ***A1*** for answer $\frac{\pi}{3}(60^\circ)$.

[7 marks]

Total [15 marks]

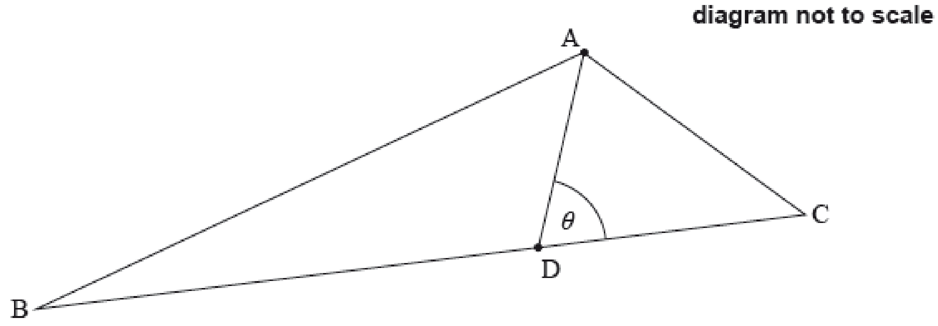
Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Let $\overrightarrow{OA} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$.

The point C is such that $\overrightarrow{AC} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$.

The following diagram shows triangle ABC. Let D be a point on [BC], with acute angle $ADC = \theta$.



a. (i) Find $\overrightarrow{AB}$. [4]

(ii) Find $\left|\overrightarrow{AB}\right|$.

b. Show that the coordinates of C are $(-2,\ 1,\ 3)$. [1]

c. Write down an expression in terms of θ for [2]

(i) angle ADB;

(ii) area of triangle ABD.

d. Given that $\frac{\text{area }\Delta\text{ABD}}{\text{area }\Delta\text{ACD}} = 3$, show that $\frac{\text{BD}}{\text{BC}} = \frac{3}{4}$. [5]

e. Hence or otherwise, find the coordinates of point D. [4]

Markscheme

a. (i) valid approach to find $\overrightarrow{AB}$

eg $\overrightarrow{OB} - \overrightarrow{OA}$, $\begin{pmatrix} 4-(-1) \\ 1-0 \\ 3-4 \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$ ***A1 N2***

(ii) valid approach to find $\left|\overrightarrow{AB}\right|$ ***(M1)***

eg $\sqrt{(5)^2 + (1)^2 + (-1)^2}$

$\left|\overrightarrow{AB}\right| = \sqrt{27}$ ***A1 N2***

[4 marks]

b. correct approach ***A1***

eg $\overrightarrow{OC} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$

C has coordinates $(-2,\ 1,\ 3)$ ***AG N0***

[1 mark]

c. (i) $\text{A}\hat{\text{D}}\text{B} = \pi - \theta, \hat{\text{D}} = 180 - \theta$ ***A1 N1***

(ii) any correct expression for the area involving θ ***A1*** ***N1***

eg $\text{area} = \frac{1}{2} \times \text{AD} \times \text{BD} \times \sin(180 - \theta),\ \frac{1}{2}ab\sin\theta,\ \frac{1}{2}\left|\overrightarrow{\text{DA}}\right|\left|\overrightarrow{\text{DB}}\right|\sin(\pi - \theta)$

[2 marks]

d. **METHOD 1** (using sine formula for area)

correct expression for the area of triangle ACD (seen anywhere) ***(A1)***

eg $\frac{1}{2}\text{AD} \times \text{DC} \times \sin\theta$

correct equation involving areas ***A1***

eg $\frac{\frac{1}{2}\text{AD}\times\text{BD}\times\sin(\pi-\theta)}{\frac{1}{2}\text{AD}\times\text{DC}\times\sin\theta} = 3$

recognizing that $\sin(\pi - \theta) = \sin\theta$ (seen anywhere) ***(A1)***

$\frac{\text{BD}}{\text{DC}} = 3$ (seen anywhere) ***(A1)***

correct approach using ratio ***A1***

eg $3\overrightarrow{\text{DC}} + \overrightarrow{\text{DC}} = \overrightarrow{\text{BC}},\ \overrightarrow{\text{BC}} = 4\overrightarrow{\text{DC}}$

correct ratio $\frac{\text{BD}}{\text{BC}} = \frac{3}{4}$ ***AG*** ***N0***

METHOD 2 (Geometric approach)

recognising ΔABD and ΔACD have same height ***(A1)***

eg use of h for both triangles, $\frac{\frac{1}{2}\text{BD}\times h}{\frac{1}{2}\text{CD}\times h} = 3$

correct approach ***A2***

eg $\text{BD} = 3x$ and $\text{DC} = x,\ \frac{\text{BD}}{\text{DC}} = 3$

correct working ***A2***

eg $\text{BC} = 4x,\ \text{BD} + \text{DC} = 4\text{DC},\ \frac{\text{BD}}{\text{BC}} = \frac{3x}{4x},\ \frac{\text{BD}}{\text{BC}} = \frac{3\text{DC}}{4\text{DC}}$

$\frac{\text{BD}}{\text{BC}} = \frac{3}{4}$ ***AG*** ***N0***

[5 marks]

e. correct working (seen anywhere) ***(A1)***

eg $\overrightarrow{\text{BD}} = \frac{3}{4}\overrightarrow{\text{BC}},\ \overrightarrow{\text{OD}} = \overrightarrow{\text{OB}} + \frac{3}{4}\begin{pmatrix}-6\\0\\0\end{pmatrix},\ \overrightarrow{\text{CD}} = \frac{1}{4}\overrightarrow{\text{CB}}$

valid approach (seen anywhere) ***(M1)***

eg $\overrightarrow{\text{OD}} = \overrightarrow{\text{OB}} + \overrightarrow{\text{BD}},\ \overrightarrow{\text{BC}} = \begin{pmatrix}-6\\0\\0\end{pmatrix}$

correct working to find x-coordinate ***(A1)***

eg $\begin{pmatrix}4\\1\\3\end{pmatrix} + \frac{3}{4}\begin{pmatrix}-6\\0\\0\end{pmatrix},\ x = 4 + \frac{3}{4}(-6),\ -2 + \frac{1}{4}(6)$

D is $\left(-\frac{1}{2},\ 1,\ 3\right)$ ***A1*** ***N3***

[4 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

A line L_1 passes though points P$(-1, 6, -1)$ and Q$(0, 4, 1)$.

A second line L_2 has equation $r = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + s\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$.

a(i) and (ii). [3]

(i) Show that $\overrightarrow{\text{PQ}} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$.

(ii) Hence, write down an equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

b. Find the cosine of the angle between $\overrightarrow{\text{PQ}}$ and L_2 . [7]

c. The lines L_1 and L_2 intersect at the point R. Find the coordinates of R. [7]

Markscheme

a(i) and (ii). (i) evidence of correct approach ***A1***

e.g. $\overrightarrow{\text{PQ}} = \overrightarrow{\text{OQ}} - \overrightarrow{\text{OP}}$, $Q - P$

$\overrightarrow{\text{PQ}} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ ***AG N0***

(ii) any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ ***A2 N2***

where $\boldsymbol{a}$ is either $\overrightarrow{\text{OP}}$ or $\overrightarrow{\text{OQ}}$ and $\boldsymbol{b}$ is a scalar multiple of $\overrightarrow{\text{PQ}}$

e.g. $\boldsymbol{r} = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$, $\boldsymbol{r} = \begin{pmatrix} t \\ 4-2t \\ 1+2t \end{pmatrix}$, $\boldsymbol{r} = 4\boldsymbol{j} + \boldsymbol{k} + t(\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k})$

[3 marks]

b. choosing a correct direction vector for L_2 ***(A1)***

e.g. $\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$

finding scalar products and magnitudes ***(A1)(A1)(A1)***

scalar product $= 1(3) - 2(0) + 2(-4)\ (= -5)$

magnitudes $= \sqrt{1^2 + (-2)^2 + 2^2}\ (= 3)$, $\sqrt{3^2 + 0^2 + (-4)^2}\ (= 5)$

substitution into formula ***M1***

e.g. $\cos\theta = \frac{-5}{\sqrt{9}\times\sqrt{25}}$

$\cos\theta = -\frac{1}{3}$ ***A2*** ***N5***

[7 marks]

c. evidence of valid approach ***(M1)***

e.g. equating lines, $L_1 = L_2$

EITHER

one correct equation in one variable ***A2***

e.g. $6 - 2t = 2$

OR

two correct equations in two variables ***A1A1***

e.g. $2t + 4s = 0$, $t - 3s = 5$

THEN

attempt to solve ***(M1)***

one correct parameter ***A1***

e.g. $t = 2$, $s = -1$

correct substitution of either parameter ***(A1)***

e.g. $\boldsymbol{r} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + (-1)\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$, $\boldsymbol{r} = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} + (+2)\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

coordinates R$(1, 2, 3)$ ***A1*** ***N3***

[7 marks]

Examiners report

a(i) and (ii). A pleasing number of candidates were successful on this straightforward vector and line question. Part (a) was generally well answered, although a few candidates still labelled their line $L =$ or used a position vector for the direction vector. Follow-through marking allowed full recovery from the latter error.

b. Few candidates wrote down their direction vector in part (b) which led to lost follow-through marks, and a common error was finding an incorrect scalar product due to difficulty multiplying by zero.

c. Part (c) was generally well understood with some candidates realizing that the equation in just one variable led to the correct parameter more quickly than solving a system of two equations to find both parameters. Some candidates gave the answer as (s, t) instead of substituting those parameters, indicating a more rote understanding of the problem. Another common error was using the same parameter for both lines. There were an alarming number of misreads of negative signs from the question or from the candidate working.

The line L_1 is parallel to the z-axis. The point P has position vector $\begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix}$ and lies on L_1.

a. Write down the equation of L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. [2]

b. The line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$. The point A has position vector $\begin{pmatrix} 6 \\ 2 \\ 9 \end{pmatrix}$. [4]

Show that A lies on L_2 .

c. Let B be the point of intersection of lines L_1 and L_2 . [7]

(i) Show that $\overrightarrow{\mathrm{OB}} = \begin{pmatrix} 8 \\ 1 \\ 14 \end{pmatrix}$.

(ii) Find $\overrightarrow{\mathrm{AB}}$.

d. The point C is at (2, 1, − 4). Let D be the point such that ABCD is a parallelogram. [3]

Find $\overrightarrow{\mathrm{OD}}$.

Markscheme

a. $L_1 : \boldsymbol{r} = \begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ ***A2 N2***

[2 marks]

b. evidence of equating $\boldsymbol{r}$ and $\overrightarrow{\mathrm{OA}}$ ***(M1)***

e.g. $\begin{pmatrix} 6 \\ 2 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$, $A = r$

one correct equation ***A1***

e.g. $6 = 2 + 2s$, $2 = 4 - s$, $9 = -1 + 5s$

$s = 2$ ***A1***

evidence of confirming for other **two** equations ***A1***

e.g. $6 = 2 + 4$, $2 = 4 - 2$, $9 = -1 + 10$

so A lies on L_2 ***AG N0***

[4 marks]

c. (i) evidence of approach ***M1***

e.g. $\begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $L_1 = L_2$

one correct equation ***A1***

e.g. $2 + 2s = 8$, $4 - s = 1$, $-1 + 5s = t$

attempt to solve ***(M1)***

finding $s = 3$ ***A1***

substituting ***M1***

e.g. $\overrightarrow{\mathrm{OB}} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + 3\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$

$\overrightarrow{OB} = \begin{pmatrix} 8 \\ 1 \\ 14 \end{pmatrix}$ ***AG N0***

(ii) evidence of appropriate approach ***(M1)***

e.g. $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$ ***A1 N2***

[7 marks]

d. evidence of appropriate approach ***(M1)***

e.g. $\overrightarrow{AB} = \overrightarrow{DC}$

correct values ***A1***

e.g. $\overrightarrow{OD} + \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$, $\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2-x \\ 1-y \\ -4-z \end{pmatrix}$

$\overrightarrow{OD} = \begin{pmatrix} 0 \\ 2 \\ -9 \end{pmatrix}$ ***A1 N2***

[3 marks]

Examiners report

a. Very few candidates gave a correct direction vector parallel to the *z*-axis. Provided they wrote down an equation here they were able to earn most of subsequent marks on follow through.

b. For (b), many found the correct parameter but neglected to confirm it in the other two equations.

c. In (c) some performed a trial and error approach to obtaining an integer parameter and thus did not "show" the mathematical origin of the result. Finding vector $\overrightarrow{AB}$ proved accessible.

d. A good number of candidates had an appropriate approach to (d), although surprisingly many subtracted $\overrightarrow{OC}$ from $\overrightarrow{AB}$ in finding $\overrightarrow{OD}$.

Consider the vectors $\boldsymbol{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

Let $2\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = 0$, where 0 is the zero vector.

. (a) Find [6]

(i) $2\boldsymbol{a} + \boldsymbol{b}$;

(ii) $|2\boldsymbol{a} + \boldsymbol{b}|$.

Let $2\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}=0$, where 0 is the zero vector.

(b) Find $\boldsymbol{c}$.

a. Find [4]

(i) $2\boldsymbol{a}+\boldsymbol{b}$;

(ii) $|2\boldsymbol{a}+\boldsymbol{b}|$.

b. Find $\boldsymbol{c}$. [2]

Markscheme

· (a) (i) $2\boldsymbol{a}=\begin{pmatrix}4\\-6\end{pmatrix}$ ***(A1)***

correct expression for $2\boldsymbol{a}+\boldsymbol{b}$ ***A1 N2***

eg $\begin{pmatrix}5\\-2\end{pmatrix}$, $(5,-2)$, $5\boldsymbol{i}-2\boldsymbol{j}$

(ii) correct substitution into length formula ***(A1)***

eg $\sqrt{5^2+2^2}$, $\sqrt{5^2+-2^2}$

$|2\boldsymbol{a}+\boldsymbol{b}|=\sqrt{29}$ ***A1 N2***

[4 marks]

(b) valid approach ***(M1)***

eg $\boldsymbol{c}=-(2\boldsymbol{a}+\boldsymbol{b})$, $5+x=0$, $-2+y=0$

$\boldsymbol{c}=\begin{pmatrix}-5\\2\end{pmatrix}$ ***A1 N2***

[2 marks]

a. (i) $2\boldsymbol{a}=\begin{pmatrix}4\\-6\end{pmatrix}$ ***(A1)***

correct expression for $2\boldsymbol{a}+\boldsymbol{b}$ ***A1 N2***

eg $\begin{pmatrix}5\\-2\end{pmatrix}$, $(5,-2)$, $5\boldsymbol{i}-2\boldsymbol{j}$

(ii) correct substitution into length formula ***(A1)***

eg $\sqrt{5^2+2^2}$, $\sqrt{5^2+-2^2}$

$|2\boldsymbol{a}+\boldsymbol{b}|=\sqrt{29}$ ***A1 N2***

[4 marks]

b. valid approach ***(M1)***

eg $\boldsymbol{c}=-(2\boldsymbol{a}+\boldsymbol{b})$, $5+x=0$, $-2+y=0$

$\boldsymbol{c}=\begin{pmatrix}-5\\2\end{pmatrix}$ ***A1 N2***

[2 marks]

Examiners report

. Most candidates comfortably applied algebraic techniques to find new vectors. However, a significant number of candidates answered part (b) as the absolute numerical value of the vector components, which suggests a misunderstanding of the modulus notation. Those who understood the notation easily made the calculation.

a. Most candidates comfortably applied algebraic techniques to find new vectors.

b. Most candidates comfortably applied algebraic techniques to find new vectors. However, a significant number of candidates answered part (b) as the absolute numerical value of the vector components, which suggests a misunderstanding of the modulus notation. Those who understood the notation easily made the calculation.

The vertices of the triangle PQR are defined by the position vectors

$\overrightarrow{\mathrm{OP}} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}$, $\overrightarrow{\mathrm{OQ}} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ and $\overrightarrow{\mathrm{OR}} = \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix}$.

a. Find [3]

(i) $\overrightarrow{\mathrm{PQ}}$;

(ii) $\overrightarrow{\mathrm{PR}}$.

b. Show that $\cos \mathrm{R\hat{P}Q} = \frac{1}{2}$. [7]

c. (i) Find $\sin \mathrm{R\hat{P}Q}$. [6]

(ii) Hence, find the area of triangle PQR, giving your answer in the form $a\sqrt{3}$.

Markscheme

a. (i) evidence of approach ***(M1)***

e.g. $\overrightarrow{\mathrm{PQ}} = \overrightarrow{\mathrm{PO}} + \overrightarrow{\mathrm{OQ}}$, $\mathrm{Q} - \mathrm{P}$

$\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$ ***A1 N2***

(ii) $\overrightarrow{\mathrm{PR}} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$ ***A1 N1***

[3 marks]

b. **METHOD 1**

choosing correct vectors $\overrightarrow{\mathrm{PQ}}$ and $\overrightarrow{\mathrm{PR}}$ ***(A1)(A1)*** finding $\overrightarrow{PQ} \bullet \overrightarrow{PR}$, $\left|\overrightarrow{PQ}\right|$, $\left|\overrightarrow{PR}\right|$ ***(A1) (A1)(A1)***

$\overrightarrow{PQ} \bullet \overrightarrow{PR} = -2 + 4 + 4 (= 6)$

$\left|\overrightarrow{PQ}\right| = \sqrt{(-1)^2 + 2^2 + 1^2} \left(= \sqrt{6}\right)$, $\left|\overrightarrow{PR}\right| = \sqrt{2^2 + 2^2 + 4^2} \left(= \sqrt{24}\right)$

substituting into formula for angle between two vectors ***M1***

e.g. $\cos R\widehat{P}Q = \frac{6}{\sqrt{6} \times \sqrt{24}}$

simplifying to expression clearly leading to $\frac{1}{2}$ ***A1***

e.g. $\frac{6}{\sqrt{6} \times 2\sqrt{6}}$, $\frac{6}{\sqrt{144}}$, $\frac{6}{12}$

$\cos R\widehat{P}Q = \frac{1}{2}$ ***AG N0***

METHOD 2

evidence of choosing cosine rule (seen anywhere) ***(M1)***

$\overrightarrow{QR} = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}$ ***A1***

$\left|\overrightarrow{QR}\right| = \sqrt{18}$, $\left|\overrightarrow{PQ}\right| = \sqrt{6}$ and $\left|\overrightarrow{PR}\right| = \sqrt{24}$ ***(A1)(A1)(A1)***

$\cos R\widehat{P}Q = \frac{\left(\sqrt{6}\right)^2 + \left(\sqrt{24}\right)^2 - \left(\sqrt{18}\right)^2}{2\sqrt{6} \times \sqrt{24}}$ ***A1***

$\cos R\widehat{P}Q = \frac{6+24-18}{24} \left(= \frac{12}{24}\right)$ ***A1***

$\cos R\widehat{P}Q = \frac{1}{2}$ ***AG N0***

[7 marks]

c. (i) **METHOD 1**

evidence of appropriate approach ***(M1)***

e.g. using $\sin^2 R\widehat{P}Q + \cos^2 R\widehat{P}Q = 1$, diagram

substituting correctly ***(A1)***

e.g. $\sin R\widehat{P}Q = \sqrt{1 - \left(\frac{1}{2}\right)^2}$

$\sin R\widehat{P}Q = \sqrt{\frac{3}{4}} \left(= \frac{\sqrt{3}}{2}\right)$ ***A1 N3***

METHOD 2

since $\cos \widehat{P} = \frac{1}{2}$, $\widehat{P} = 60^\circ$ ***(A1)***

evidence of approach

e.g. drawing a right triangle, finding the missing side ***(A1)***

$\sin \widehat{P} = \frac{\sqrt{3}}{2}$ ***A1 N3***

(ii) evidence of appropriate approach ***(M1)***

e.g. attempt to substitute into $\frac{1}{2}ab\sin C$

correct substitution

e.g. area $= \frac{1}{2}\sqrt{6} \times \sqrt{24} \times \frac{\sqrt{3}}{2}$ ***A1***

area $= 3\sqrt{3}$ ***A1 N2***

[6 marks]

Examiners report

a. Combining the vectors in (a) was generally well done, although some candidates reversed the subtraction, while others calculated the magnitudes.

b. Many candidates successfully used scalar product and magnitude calculations to complete part (b). Alternatively, some used the cosine rule, and often achieved correct results. Some assumed the triangle was a right-angled triangle and thus did not earn full marks. Although PQR is indeed right-angled, in a "show that" question this attribute must be directly established.

c. Many candidates attained the value for sine in (c) with little difficulty, some using the Pythagorean identity, while others knew the side relationships in a 30-60-90 triangle. Unfortunately, a good number of candidates then used the side values of $1, 2, \sqrt{3}$ to find the area of PQR , instead of the magnitudes of the vectors found in (a). Furthermore, the "hence" command was sometimes neglected as the value of sine was expected to be used in the approach.

Let A and B be points such that $\overrightarrow{\mathrm{OA}} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$ and $\overrightarrow{\mathrm{OB}} = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix}$.

a. Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$. [1]

b. Let C and D be points such that ABCD is a **rectangle**. [4]

Given that $\overrightarrow{\mathrm{AD}} = \begin{pmatrix} 4 \\ p \\ 1 \end{pmatrix}$, show that $p = 3$.

c. Let C and D be points such that ABCD is a **rectangle**. [4]

Find the coordinates of point C.

d. Let C and D be points such that ABCD is a **rectangle**. [5]

Find the area of rectangle ABCD.

Markscheme

a. correct approach **A1**

e.g. $\overrightarrow{\mathrm{AO}}+\overrightarrow{\mathrm{OB}}$, $\begin{pmatrix}6\\0\\3\end{pmatrix}-\begin{pmatrix}5\\2\\1\end{pmatrix}$

$\overrightarrow{\mathrm{AB}}=\begin{pmatrix}1\\-2\\2\end{pmatrix}$ ***AG*** ***N0***

[1 mark]

b. recognizing $\overrightarrow{\mathrm{AD}}$ is perpendicular to $\overrightarrow{\mathrm{AB}}$ (may be seen in sketch) ***(R1)***

e.g. adjacent sides of rectangle are perpendicular

recognizing dot product must be zero ***(R1)***

e.g. $\overrightarrow{\mathrm{AD}}\bullet\overrightarrow{AB}=0$

correct substitution ***(A1)***

e.g. $(1\times 4)+(-2\times p)+(2\times 1)$, $4-2p+2=0$

equation which clearly leads to $p=3$ ***A1***

e.g. $6-2p=0$, $2p=6$

$p=3$ ***AG*** ***N0***

[4 marks]

c. correct approach (seen anywhere including sketch) ***(A1)***

e.g. $\overrightarrow{\mathrm{OC}}=\overrightarrow{\mathrm{OB}}+\overrightarrow{\mathrm{BC}}$, $\overrightarrow{\mathrm{OD}}+\overrightarrow{\mathrm{DC}}$

recognizing opposite sides are equal vectors (may be seen in sketch) ***(R1)***

e.g. $\overrightarrow{\mathrm{BC}}=\overrightarrow{\mathrm{AD}}$, $\overrightarrow{\mathrm{DC}}=\overrightarrow{\mathrm{AB}}$, $\begin{pmatrix}6\\0\\3\end{pmatrix}+\begin{pmatrix}4\\3\\1\end{pmatrix}$, $\begin{pmatrix}9\\5\\2\end{pmatrix}+\begin{pmatrix}1\\-2\\2\end{pmatrix}$

coordinates of point C are (10, 3, 4) (accept $\begin{pmatrix}10\\3\\4\end{pmatrix}$) ***A2*** ***N4***

Note: Award ***A1*** for two correct values.

[4 marks]

d. attempt to find one side of the rectangle ***(M1)***

e.g. substituting into magnitude formula

two correct magnitudes ***A1A1***

e.g. $\sqrt{(1)^2+(-2)^2+2^2}$, 3 ; $\sqrt{16+9+1}$, $\sqrt{26}$

multiplying magnitudes ***(M1)***

e.g. $\sqrt{26}\times\sqrt{9}$

area $=\sqrt{234}(=3\sqrt{26})$ (accept $3\times\sqrt{26}$) ***A1*** ***N3***

[5 marks]

Examiners report

a. Part (a) was answered correctly by nearly every candidate.

b. In part (b), the candidates who realized that the vectors must be perpendicular were successful using the scalar product to find p . Incorrect approaches included using magnitudes, or creating vector equations of lines for the sides and setting them equal to each other. In addition, there were a good number of candidates who worked backwards, using the given value of 3 for p to find the coordinates of point D. Candidates who work backwards on a "show that" question will earn no marks.

c. Part (c) was more difficult for candidates, and was left blank by some. Some candidates found $\overrightarrow{\mathrm{AC}}$ rather than $\overrightarrow{\mathrm{OC}}$, as required. Many candidates recognized that the opposite sides of the rectangle must be equal, but did not consider the directions of the vectors for those sides. There were also a good number of candidates who mislabelled the vertices of their rectangles, which led to them working with a rectangle ABDC, rather than ABCD.

d. The majority of candidates who attempted part (d) were successful in multiplying the magnitudes of the sides. Unfortunately, there were some who set up their solutions correctly, then had arithmetic errors in their working.

Distances in this question are in metres.

Ryan and Jack have model airplanes, which take off from level ground. Jack’s airplane takes off after Ryan’s.

The position of Ryan’s airplane t seconds after it takes off is given by $\boldsymbol{r} = \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} + t\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$.

a. Find the speed of Ryan’s airplane. [3]

b. Find the height of Ryan’s airplane after two seconds. [2]

c. The position of Jack’s airplane s seconds after **it** takes off is given by $\boldsymbol{r} = \begin{pmatrix} -39 \\ 44 \\ 0 \end{pmatrix} + s\begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$. [5]

Show that the paths of the airplanes are perpendicular.

d. The two airplanes collide at the point $(-23, 20, 28)$. [5]

How long after Ryan’s airplane takes off does Jack’s airplane take off?

Markscheme

a. valid approach ***(M1)***

eg magnitude of direction vector

correct working ***(A1)***

eg $\sqrt{(-4)^2 + 2^2 + 4^2}$, $\sqrt{-4^2 + 2^2 + 4^2}$

$6\ (\mathrm{ms}^{-1})$ ***A1 N2***

[3 marks]

b. substituting 2 for t ***(A1)***

eg $0+2(4), \boldsymbol{r} = \begin{pmatrix}5\\6\\0\end{pmatrix} + 2\begin{pmatrix}-4\\2\\4\end{pmatrix}, \begin{pmatrix}-3\\10\\8\end{pmatrix}, y=10$

8 (metres) ***A1*** ***N2***

[2 marks]

c. **METHOD 1**

choosing correct direction vectors $\begin{pmatrix}-4\\2\\4\end{pmatrix}$ and $\begin{pmatrix}4\\-6\\7\end{pmatrix}$ ***(A1)(A1)***

evidence of scalar product ***M1***

eg $\boldsymbol{a} \cdot \boldsymbol{b}$

correct substitution into scalar product ***(A1)***

eg $(-4 \times 4) + (2 \times -6) + (4 \times 7)$

evidence of correct calculation of the scalar product as 0 ***A1***

eg $-16 - 12 + 28 = 0$

directions are perpendicular ***AG*** ***N0***

METHOD 2

choosing correct direction vectors $\begin{pmatrix}-4\\2\\4\end{pmatrix}$ and $\begin{pmatrix}4\\-6\\7\end{pmatrix}$ ***(A1)(A1)***

attempt to find angle between vectors ***M1***

correct substitution into numerator ***A1***

eg $\cos\theta = \frac{-16-12+28}{|a||b|}, \ \cos\theta = 0$

$\theta = 90^\circ$ ***A1***

directions are perpendicular ***AG*** ***N0***

[5 marks]

d. **METHOD 1**

one correct equation for Ryan's airplane ***(A1)***

eg $5-4t=-23, \ 6+2t=20, \ 0+4t=28$

$t=7$ ***A1***

one correct equation for Jack's airplane ***(A1)***

eg $-39+4s=-23, \ 44-6s=20, \ 0+7s=28$

$s=4$ ***A1***

3 (seconds later) ***A1*** ***N2***

METHOD 2

valid approach ***(M1)***

eg $\begin{pmatrix}5\\6\\0\end{pmatrix} + t\begin{pmatrix}-4\\2\\4\end{pmatrix} = \begin{pmatrix}-39\\44\\0\end{pmatrix} + s\begin{pmatrix}4\\-6\\7\end{pmatrix}$, one correct equation

two correct equations ***(A1)***

eg $5-4t=-39+4s, \ 6+2t=44-6s, \ 4t=7s$

$t=7$ ***A1***

$s=4$ ***A1***

3 (seconds later) ***A1*** ***N2***

[5 marks]

Examiners report

a. [N/A]
[N/A]

b. [N/A]
d. [N/A]

The line L_1 passes through the points P(2, 4, 8) and Q(4, 5, 4) .

The line L_2 is perpendicular to L_1 , and parallel to $\begin{pmatrix} 3p \\ 2p \\ 4 \end{pmatrix}$, where $p \in \mathbb{Z}$.

a(i) and (ii). (i) Find $\overrightarrow{\text{PQ}}$. [4]

(ii) Hence write down a vector equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + s\boldsymbol{b}$.

b(i) and (ii). (i) Find the value of p . [7]

(ii) Given that L_2 passes through $\text{R}(10, 6, -40)$, write down a vector equation for L_2 .

c. The lines L_1 and L_2 intersect at the point A. Find the x-coordinate of A. [7]

Markscheme

a(i) and (ii). (i) evidence of approach ***(M1)***

e.g. $\overrightarrow{\text{PO}} + \overrightarrow{\text{OQ}}$, P – Q

$\overrightarrow{\text{PQ}} = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$ ***A1 N2***

(ii) **any** correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + s\boldsymbol{b}$ (accept any parameter for s)

where $\boldsymbol{a}$ is $\begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix}$ or $\begin{pmatrix} 4 \\ 5 \\ 4 \end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$ ***A2 N2***

e.g. $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix} + s\begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$, $\boldsymbol{r} = \begin{pmatrix} 4+2s \\ 5+1s \\ 4-4s \end{pmatrix}$, $\boldsymbol{r} = 2\boldsymbol{i} + 4\boldsymbol{j} + 8\boldsymbol{k} + s(2\boldsymbol{i} + 1\boldsymbol{j} - 4\boldsymbol{k})$

Note: Award ***A1*** for the form $\boldsymbol{a} + s\boldsymbol{b}$, ***A1*** for $\boldsymbol{L} = \boldsymbol{a} + s\boldsymbol{b}$, ***A0*** for $\boldsymbol{r} = \boldsymbol{b} + s\boldsymbol{a}$.

[4 marks]

b(i) and (ii). (i) choosing correct direction vectors for L_1 and L_2 ***(A1) (A1)***

e.g. $\begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$, $\begin{pmatrix} 3p \\ 2p \\ 4 \end{pmatrix}$

evidence of equating scalar product to 0 ***(M1)***

correct calculation of scalar product ***A1***

e.g. $2 \times 3p + 1 \times 2p + (-4) \times 4$, $8p - 16 = 0$

$p = 2$ ***A1 N3***

(ii) **any** correct expression in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a}$ is $\begin{pmatrix} 10 \\ 6 \\ -40 \end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix} 6 \\ 4 \\ 4 \end{pmatrix}$ ***A2 N2***

e.g. $\boldsymbol{r} = \begin{pmatrix} 10 \\ 6 \\ -40 \end{pmatrix} + t\begin{pmatrix} 6 \\ 4 \\ 4 \end{pmatrix}$, $\boldsymbol{r} = \begin{pmatrix} 10+6s \\ 6+4s \\ -40+4s \end{pmatrix}$, $\boldsymbol{r} = 10\boldsymbol{i} + 6\boldsymbol{j} - 40\boldsymbol{k} + s(6\boldsymbol{i} + 4\boldsymbol{j} + 4\boldsymbol{k})$

Note: Award ***A1*** for the form $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** for $\boldsymbol{L} = \boldsymbol{a} + t\boldsymbol{b}$ (unless they have been penalised for $\boldsymbol{L} = \boldsymbol{a} + s\boldsymbol{b}$ in part (a)), ***A0*** for $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[7 marks]

c. appropriate approach ***(M1)***

e.g. $\begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix} + s\begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 10 \\ 6 \\ -40 \end{pmatrix} + t\begin{pmatrix} 6 \\ 4 \\ 4 \end{pmatrix}$

any two correct equations with **different** parameters ***A1A1***

e.g. $2 + 2s = 10 + 6t$, $4 + s = 6 + 4t$, $8 - 4s = -40 + 4t$

attempt to solve simultaneous equations ***(M1)***

correct working ***(A1)***

e.g. $-6 = -2 - 2t$, $4 = 2t$, $-4 + 5s = 46$, $5s = 50$

one correct parameter $s = 10$, $t = 2$ ***A1***

$x = 22$ (accept (22, 14, −32)) ***A1 N4***

[7 marks]

Examiners report

a(i) and (ii). In part (a), nearly all the candidates correctly found the vector PQ, and the majority went onto find the correct vector equation of the line.

There are still many candidates who do not write this equation in the correct form, using "$\boldsymbol{r}$ = ", and these candidates were penalized one mark.

b(i) and (ii). In part (b), the majority of candidates knew to set the scalar product equal to zero for the perpendicular vectors, and were able to find the correct value of p.

c. A good number of candidates used the correct method to find the intersection of the two lines, though some algebraic and arithmetic errors kept some from finding the correct final answer.

A line L passes through points $\mathrm{A}(-3,\ 4,\ 2)$ and $\mathrm{B}(-1,\ 3,\ 3)$.

The line L also passes through the point $\mathrm{C}(3,\ 1,\ p)$.

a.i. Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$. [1]

a.ii. Find a vector equation for L. [2]

b. Find the value of p. [5]

c. The point D has coordinates $(q^2,\ 0,\ q)$. Given that $\overrightarrow{\mathrm{DC}}$ is perpendicular to L, find the possible values of q. [7]

Markscheme

a.i. correct approach ***A1***

eg $\begin{pmatrix}-1\\3\\3\end{pmatrix} - \begin{pmatrix}-3\\4\\2\end{pmatrix},\ \begin{pmatrix}3\\-4\\-2\end{pmatrix} + \begin{pmatrix}-1\\3\\3\end{pmatrix}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix}2\\-1\\1\end{pmatrix}$ ***AG N0***

[1 mark]

a.ii. any correct equation in the form $r = a + tb$ (any parameter for t)

where a is $\begin{pmatrix}-3\\4\\2\end{pmatrix}$ or $\begin{pmatrix}-1\\3\\3\end{pmatrix}$ and b is a scalar multiple of $\begin{pmatrix}2\\-1\\1\end{pmatrix}$ ***A2 N2***

eg $r = \begin{pmatrix}-3\\4\\2\end{pmatrix} + t\begin{pmatrix}2\\-1\\1\end{pmatrix},\ (x,\ y,\ z) = (-1,\ 3,\ 3) + s(-2,\ 1,\ -1),\ r = \begin{pmatrix}-3+2t\\4-t\\2+t\end{pmatrix}$

Note: Award ***A1*** for the form $a + tb$, **A1** for the form $L = a + tb$, **A0** for the form $r = b + ta$.

[2 marks]

b. **METHOD 1 – finding value of parameter**

valid approach ***(M1)***

eg $\begin{pmatrix}-3\\4\\2\end{pmatrix} + t\begin{pmatrix}2\\-1\\1\end{pmatrix} = \begin{pmatrix}3\\1\\p\end{pmatrix},\ (-1,\ 3,\ 3) + s(-2,\ 1,\ -1) = (3,\ 1,\ p)$

one correct equation (not involving p) ***(A1)***

eg $-3+2t = 3,\ -1-2s = 3,\ 4-t = 1,\ 3+s = 1$

correct parameter from their equation (may be seen in substitution) ***A1***

eg $t = 3,\ s = -2$

correct substitution ***(A1)***

eg $\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + 3\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ p \end{pmatrix}$, $3-(-2)$

$p=5$ $\left(\text{accept } \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}\right)$ ***A1*** ***N2***

METHOD 2 – eliminating parameter

valid approach ***(M1)***

eg $\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ p \end{pmatrix}$, $(-1,\ 3,\ 3) + s(-2,\ 1,\ -1) = (3,\ 1,\ p)$

one correct equation (not involving p) ***(A1)***

eg $-3+2t=3,\ -1-2s=3,\ 4-t=1,\ 3+s=1$

correct equation (with p) ***A1***

eg $2+t=p,\ 3-s=p$

correct working to solve for p ***(A1)***

eg $7=2p-3,\ 6=1+p$

$p=5$ $\left(\text{accept } \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}\right)$ ***A1*** ***N2***

[5 marks]

c. valid approach to find $\overrightarrow{\mathrm{DC}}$ or $\overrightarrow{\mathrm{CD}}$ ***(M1)***

eg $\begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix} - \begin{pmatrix} q^2 \\ 0 \\ q \end{pmatrix}$, $\begin{pmatrix} q^2 \\ 0 \\ q \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}$, $\begin{pmatrix} q^2 \\ 0 \\ q \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \\ p \end{pmatrix}$

correct vector for $\overrightarrow{\mathrm{DC}}$ or $\overrightarrow{\mathrm{CD}}$ (may be seen in scalar product) ***A1***

eg $\begin{pmatrix} 3-q^2 \\ 1 \\ 5-q \end{pmatrix}$, $\begin{pmatrix} q^2-3 \\ -1 \\ q-5 \end{pmatrix}$, $\begin{pmatrix} 3-q^2 \\ 1 \\ p-q \end{pmatrix}$

recognizing scalar product of $\overrightarrow{\mathrm{DC}}$ or $\overrightarrow{\mathrm{CD}}$ with direction vector of L is zero (seen anywhere) ***(M1)*** *eg* $\begin{pmatrix} 3-q^2 \\ 1 \\ p-q \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = 0,\ \overrightarrow{\mathrm{DC}} \bullet \overrightarrow{\mathrm{AC}} = 0,\ \begin{pmatrix} 3-q^2 \\ 1 \\ 5-q \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = 0$

correct scalar product in terms of only q ***A1***

eg $6-2q^2-1+5-q,\ 2q^2+q-10=0,\ 2(3-q^2)-1+5-q$

correct working to solve quadratic ***(A1)***

eg $(2q+5)(q-2),\ \frac{-1\pm\sqrt{1-4(2)(-10)}}{2(2)}$

$q=-\frac{5}{2},\ 2$ ***A1A1*** ***N3***

[7 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]
c. [N/A]

The line L is parallel to the vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

The line L passes through the point $(9, 4)$.

a. Find the gradient of the line L. [2]

b. Find the equation of the line L in the form $y = ax + b$. [3]

c. Write down a vector equation for the line L. [2]

Markscheme

a. attempt to find gradient ***(M1)***

eg reference to change in x is 3 and/or y is 2, $\frac{3}{2}$

gradient $= \frac{2}{3}$ ***A1*** ***N2***

[2 marks]

b. attempt to substitute coordinates and/or gradient into Cartesian equation for a line ***(M1)***

eg $y-4=m(x-9),\ y=\frac{2}{3}x+b,\ 9=a(4)+c$

correct substitution ***(A1)***

eg $4=\frac{2}{3}(9)+c,\ y-4=\frac{2}{3}(x-9)$

$y=\frac{2}{3}x-2\ \left(\text{accept } a=\frac{2}{3},\ b=-2\right)$ ***A1*** ***N2***

[3 marks]

c. **any** correct equation in the form ***r*** = ***a*** + *t**b*** (any parameter for *t*), where ***a*** indicates position *eg* $\begin{pmatrix} 9 \\ 4 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$, and ***b*** is a scalar multiple of $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

eg $\boldsymbol{r} = \begin{pmatrix} 9 \\ 4 \end{pmatrix} + t\begin{pmatrix} 3 \\ 2 \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3t+9 \\ 2t+4 \end{pmatrix}, \boldsymbol{r} = 0i - 2j + s(3i + 2j)$ ***A2*** ***N2***

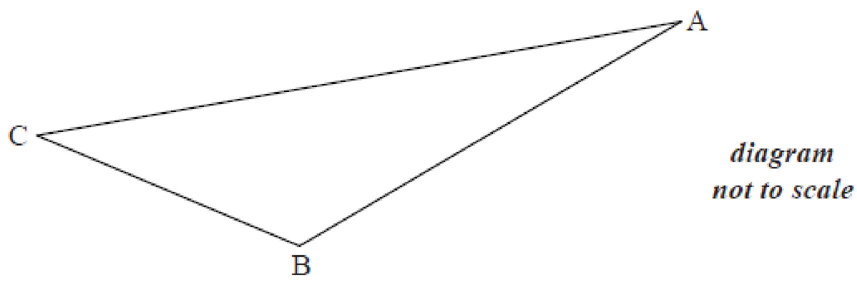
Note: Award ***A1*** for ***a*** + *t**b***, ***A1*** for *L* = ***a*** + *t**b***, ***A0*** for ***r*** = ***b*** + *t**a***.

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The following diagram shows the obtuse-angled triangle ABC such that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} -3 \\ 0 \\ -4 \end{pmatrix}$ and $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} -2 \\ 2 \\ -6 \end{pmatrix}$.



a(i) and (ii). (i) Write down $\overrightarrow{\mathrm{BA}}$. [3]

(ii) Find $\overrightarrow{\mathrm{BC}}$.

b(i) and (ii). (i) Find $\cos \mathrm{A\hat{B}C}$. [7]

(ii) Hence, find $\sin \mathrm{A\hat{B}C}$.

c(i) and (ii). [6]

The point D is such that $\overrightarrow{\mathrm{CD}} = \begin{pmatrix} -4 \\ 5 \\ p \end{pmatrix}$, where $p > 0$.

(i) Given that $|\overrightarrow{\mathrm{CD}}| = \sqrt{50}$, show that $p = 3$.

(ii) Hence, show that $\overrightarrow{\mathrm{CD}}$ is perpendicular to $\overrightarrow{\mathrm{BC}}$.

Markscheme

a(i) and (ii).

(i) $\overrightarrow{\mathrm{BA}} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$ ***A1 N1***

(ii) evidence of combining vectors ***(M1)***

e.g. $\overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BC}} = \overrightarrow{\mathrm{AC}}$, $\overrightarrow{\mathrm{BA}} + \overrightarrow{\mathrm{AC}}$, $\begin{pmatrix} -2 \\ 2 \\ -6 \end{pmatrix} - \begin{pmatrix} -3 \\ 0 \\ -4 \end{pmatrix}$

$\overrightarrow{\mathrm{BC}} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ ***A1 N2***

[3 marks]

b(i) and (ii).

(i) **METHOD 1**

finding $\overrightarrow{\mathrm{BA}} \bullet \overrightarrow{\mathrm{BC}}$, $|\overrightarrow{\mathrm{BA}}|$, $|\overrightarrow{\mathrm{BC}}|$

e.g. $\overrightarrow{\mathrm{BA}} \bullet \overrightarrow{\mathrm{BC}} = 3 \times 1 + 0 + 4 \times -2$, $|\overrightarrow{\mathrm{BA}}| = \sqrt{3^2 + 4^2}$, $|\overrightarrow{\mathrm{BC}}| = 3$

substituting into formula for $\cos\theta$ ***M1***

e.g. $\frac{3\times1+0+4\times-2}{3\sqrt{3^2+0+4^2}}$, $\frac{-5}{5\times3}$

$cos \mathrm{A\hat{B}C} = -\frac{5}{15} \left(= -\frac{1}{3}\right)$ ***A1 N3***

METHOD 2

finding $|\overrightarrow{\mathrm{AC}}|$, $|\overrightarrow{\mathrm{BA}}|$, $|\overrightarrow{\mathrm{BC}}|$ ***(A1)(A1)(A1)***

e.g. $|\overrightarrow{\mathrm{AC}}| = \sqrt{2^2 + 2^2 + 6^2}$, $|\overrightarrow{\mathrm{BA}}| = \sqrt{3^2 + 4^2}$, $|\overrightarrow{\mathrm{BC}}| = 3$

substituting into cosine rule ***M1***

e.g. $\frac{5^2+3^2-\left(\sqrt{44}\right)^2}{2\times5\times3}$, $\frac{25+9-44}{30}$

$\cos \mathrm{A\hat{B}C} = -\frac{10}{30} \left(= -\frac{1}{3}\right)$ ***A1 N3***

(ii) evidence of using Pythagoras ***(M1)***

e.g. right-angled triangle with values, $\sin^2 x + \cos^2 x = 1$

$\sin \mathrm{A\hat{B}C} = \frac{\sqrt{8}}{3} \left(= \frac{2\sqrt{2}}{3}\right)$ ***A1 N2***

[7 marks]

c(i) and (ii).

(i) attempt to find an expression for $|\overrightarrow{\mathrm{CD}}|$ ***(M1)***

e.g. $\sqrt{(-4)^2 + 5^2 + p^2}$, $|\overrightarrow{\mathrm{CD}}|^2 = 4^2 + 5^2 + p^2$

correct equation ***A1***

e.g. $\sqrt{(-4)^2 + 5^2 + p^2} = \sqrt{50}$, $4^2 + 5^2 + p^2 = 50$

$p^2 = 9$ ***A1***

$p = 3$ ***AG N0***

(ii) evidence of scalar product ***(M1)***

e.g. $\begin{pmatrix} -4 \\ 5 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$, $\overrightarrow{\mathrm{CD}} \bullet \overrightarrow{\mathrm{BC}}$

correct substitution

e.g. $-4 \times 1 + 5 \times 2 + 3 \times -2$, $-4 + 10 - 6$ ***A1***

$\overrightarrow{CD} \bullet \overrightarrow{BC} = 0$ ***A1***

$\overrightarrow{CD}$ is perpendicular to $\overrightarrow{BC}$ ***AG*** ***N0***

[6 marks]

Examiners report

a(i) and (ii). Many candidates answered (a) correctly, although some reversed the vectors when finding $\overrightarrow{BC}$, while others miscopied the vectors from the question paper.

b(i) and (ii). Students had no difficulty finding the scalar product and magnitudes of the vectors used in finding the cosine. However, few recognized that $\overrightarrow{BA}$ is the vector to apply in the formula to find the cosine value. Most used $\overrightarrow{AB}$ to obtain a positive cosine, which neglects that the angle is obtuse and thus has a negative cosine. Surprisingly few students could then take a value for cosine and use it to find a value for sine. Most left (bii) blank entirely.

c(i) and (ii). Part (c) proved accessible for many candidates. Some created an expression for $|\overrightarrow{CD}|$ and then substituted the given $p = 3$ to obtain $\sqrt{50}$, which does not satisfy the "show that" instruction. Many students recognized that the scalar product must be zero for vectors to be perpendicular, and most provided the supporting calculations.

The line L passes through the point $(5, -4, 10)$ and is parallel to the vector $\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$.

a. Write down a vector equation for line L . [2]

b. The line L intersects the x-axis at the point P. Find the x-coordinate of P. [6]

Markscheme

a. **any** correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a}$ is $\begin{pmatrix} 5 \\ -4 \\ 10 \end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$ ***A2*** ***N2***

e.g. $\boldsymbol{r} = \begin{pmatrix} 5 \\ -4 \\ 10 \end{pmatrix} + t\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$, $\boldsymbol{r} = 5\boldsymbol{i} - 4\boldsymbol{j} + 10\boldsymbol{k} + t(-8\boldsymbol{i} + 4\boldsymbol{j} - 10\boldsymbol{k})$

Note: Award ***A1*** for the form $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** for $L = \boldsymbol{a} + t\boldsymbol{b}$, ***A0*** for $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[2 marks]

b. recognizing that $y = 0$ or $z = 0$ at x-intercept (seen anywhere) ***(R1)***

attempt to set up equation for x-intercept (must suggest $x \neq 0$) ***(M1)***

e.g. $L = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}$, $5 + 4t = x$, $r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

one correct equation in one variable ***(A1)***

e.g. $-4 - 2t = 0$, $10 + 5t = 0$

finding $t = -2$ ***A1***

correct working ***(A1)***

e.g. $x = 5 + (-2)(4)$

$x = -3$ (accept $(-3, 0, 0)$) ***A1*** ***N3***

[6 marks]

Examiners report

a. In part (a), the majority of candidates correctly recognized the equation that contains the position and direction vectors of a line. However, we saw a large number of candidates who continue to write their equations using "$L =$ ", rather than the mathematically correct "$\boldsymbol{r} =$ " or " $\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$". $\boldsymbol{r}$ and $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ represent vectors, whereas L is simply the name of the line. For part (b), very few candidates recognized that a general point on the x-axis will be given by the vector $\begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}$. Common errors included candidates setting their equation equal to $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, or $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, or even just the number 0.

b. In part (a), the majority of candidates correctly recognized the equation that contains the position and direction vectors of a line. However, we saw a large number of candidates who continue to write their equations using "$L =$ ", rather than the mathematically correct "$\boldsymbol{r} =$ " or " $\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$". $\boldsymbol{r}$ and $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ represent vectors, whereas L is simply the name of the line. For part (b), very few candidates recognized that a general point on the x-axis will be given by the vector $\begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}$. Common errors included candidates setting their equation equal to $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, or $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, or even just the number 0.

A particle is moving with a constant velocity along line L . Its initial position is A(6 , −2 , 10) . After one second the particle has moved to B(9, −6 , 15) .

a(i) and (ii). (i) Find the velocity vector, $\overrightarrow{\text{AB}}$. [4]

(ii) Find the speed of the particle.

b. Write down an equation of the line L . [2]

Markscheme

a(i) and (ii). (i) evidence of approach *(M1)*

e.g. $\overrightarrow{\mathrm{AO}}+\overrightarrow{\mathrm{OB}}$, $\mathrm{B}-\mathrm{A}$, $\begin{pmatrix} 9-6 \\ -6+2 \\ 15-10 \end{pmatrix}$

$\overrightarrow{\mathrm{AB}}=\begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}$ (accept $(3,-4,5)$) ***A1 N2***

(ii) evidence of finding the magnitude of the velocity vector ***M1***

e.g. speed $=\sqrt{3^2+4^2+5^2}$

speed $=\sqrt{50}\left(=5\sqrt{2}\right)$ ***A1 N1***

[4 marks]

b. correct **equation** (accept Cartesian and parametric forms) ***A2 N2***

e.g. $\boldsymbol{r}=\begin{pmatrix} 6 \\ -2 \\ 10 \end{pmatrix}+t\begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}$, $\boldsymbol{r}=\begin{pmatrix} 9 \\ -6 \\ 15 \end{pmatrix}+t\begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}$

[2 marks]

Examiners report

a(i) and (ii). This question was quite well done. Marks were lost when candidates found the vector $\overrightarrow{\mathrm{BA}}$ instead of $\overrightarrow{\mathrm{AB}}$ in part (a) and for not writing their vector equation as an equation.

b. In part (b), a few candidates switched the position and velocity vectors or used the vectors $\overrightarrow{\mathrm{OA}}$ and $\overrightarrow{\mathrm{OB}}$ to incorrectly write the vector equation.

The line L_1 is represented by the vector equation $\boldsymbol{r}=\begin{pmatrix} -3 \\ -1 \\ -25 \end{pmatrix}+p\begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}$.

A second line L_2 is parallel to L_1 and passes through the point $\mathrm{B}(-8,-5,25)$.

a. Write down a vector equation for L_2 in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$. [2]

b. A third line L_3 is perpendicular to L_1 and is represented by $\boldsymbol{r}=\begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix}+q\begin{pmatrix} -7 \\ -2 \\ k \end{pmatrix}$. [5]

Show that $k=-2$.

c. The lines L_1 and L_3 intersect at the point A. [6]

Find the coordinates of A.

d(i) and (ii). [5]

The lines L_2 and L_3 intersect at point C where $\overrightarrow{BC} = \begin{pmatrix} 6 \\ 3 \\ -24 \end{pmatrix}$.

(i) Find $\overrightarrow{AB}$.

(ii) Hence, find $|\overrightarrow{AC}|$.

Markscheme

a. any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (accept any parameter) ***A2 N2***

e.g. $\boldsymbol{r} = \begin{pmatrix} -8 \\ -5 \\ 25 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}$

Note: Award ***A1*** for $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** for $L = \boldsymbol{a} + t\boldsymbol{b}$, ***A0*** for $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[2 marks]

b. recognizing scalar product must be zero (seen anywhere) ***R1***

e.g. $\boldsymbol{a} \bullet \boldsymbol{b} = 0$

evidence of choosing direction vectors $\begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}, \begin{pmatrix} -7 \\ -2 \\ k \end{pmatrix}$ ***(A1)(A1)***

correct calculation of scalar product ***(A1)***

e.g. $2(-7) + 1(-2) - 8k$

simplification that clearly leads to solution ***A1***

e.g. $-16 - 8k$, $-16 - 8k = 0$

$k = -2$ ***AG N0***

[5 marks]

c. evidence of equating vectors ***(M1)***

e.g. $L_1 = L_3$, $\begin{pmatrix} -3 \\ -1 \\ -25 \end{pmatrix} + p\begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} + q\begin{pmatrix} -7 \\ -2 \\ -2 \end{pmatrix}$

any **two** correct equations ***A1A1***

e.g. $-3 + 2p = 5 - 7q$, $-1 + p = -2q$, $-25 - 8p = 3 - 2q$

attempting to solve equations ***(M1)***

finding **one** correct parameter ($p = -3$, $q = 2$) ***A1***

the coordinates of A are $(-9, -4, -1)$ ***A1 N3***

[6 marks]

d(i) and (ii). (i) evidence of appropriate approach ***(M1)*** e.g. $\overrightarrow{\mathrm{OA}} + \overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}}$, $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} -8 \\ -5 \\ 25 \end{pmatrix} - \begin{pmatrix} -9 \\ -4 \\ -1 \end{pmatrix}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 1 \\ -1 \\ 26 \end{pmatrix}$ ***A1 N2***

(ii) finding $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 7 \\ 2 \\ 2 \end{pmatrix}$ ***A1***

evidence of finding magnitude ***(M1)***

e.g. $|\overrightarrow{\mathrm{AC}}| = \sqrt{7^2 + 2^2 + 2^2}$

$|\overrightarrow{\mathrm{AC}}| = \sqrt{57}$ ***A1 N3***

[5 marks]

Examiners report

a. Many candidates gave a correct vector equation for the line.

b. A common error was to misplace the initial position and direction vectors. Those who set the scalar product of the direction vectors to zero typically solved for k successfully. Those who substituted $k = -2$ earned fewer marks for working backwards in a "show that" question.

c. Many went on to find the coordinates of point A, however some used the same letter, say p, for each parameter and thus could not solve the system.

d(i). Part (i) and (ii) proved challenging as many candidates did not consider that $\overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BC}} = \overrightarrow{\mathrm{AC}}$. Rather, many attempted to find the coordinates of point C, which became a more arduous and error-prone task.

The position vectors of points P and Q are $\boldsymbol{i} + 2\,\boldsymbol{j} - \boldsymbol{k}$ and $7\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$ respectively.

a. Find a vector equation of the line that passes through P and Q. [4]

b. The line through P and Q is perpendicular to the vector $2\boldsymbol{i} + n\boldsymbol{k}$. Find the value of n. [3]

Markscheme

a. valid attempt to find direction vector ***(M1)***

eg $\overrightarrow{\mathrm{PQ}}$, $\overrightarrow{\mathrm{QP}}$

correct direction vector (or multiple of) ***(A1)***

eg $6\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$

any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (any parameter for t) ***A2 N3***

where $\boldsymbol{a}$ is $\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ or $7\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$, and $\boldsymbol{b}$ is a scalar multiple of $6\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$

eg $\boldsymbol{r} = 7\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k} + t(6\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k})$, $\boldsymbol{r} = \begin{pmatrix} 1+6s \\ 2+1s \\ -1-3s \end{pmatrix}$, $r = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + t\begin{pmatrix} -6 \\ -1 \\ 3 \end{pmatrix}$

Notes: Award ***A1*** for the form $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** for the form $\boldsymbol{L} = \boldsymbol{a} + t\boldsymbol{b}$, ***A0*** for the form $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[4 marks]

b. correct expression for scalar product ***(A1)***

eg $6 \times 2 + 1 \times 0 + (-3) \times n, \ -3n + 12$

setting scalar product equal to zero (seen anywhere) ***(M1)***

eg $\boldsymbol{u} \bullet \boldsymbol{v} = 0, \ -3n + 12 = 0$

$n = 4$ ***A1*** ***N2***

[3 marks]

Examiners report

a. [N/A]

b. [N/A]

A line L passes through $\mathrm{A}(1, -1, 2)$ and is parallel to the line $\boldsymbol{r} = \begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix} + s\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$.

The line L passes through point P when $t = 2$.

a. Write down a vector equation for L in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. [2]

b(i) and (ii). Find [4]

(i) $\overrightarrow{\mathrm{OP}}$;

(ii) $|\overrightarrow{\mathrm{OP}}|$.

Markscheme

a. correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ ***A2*** ***N2***

$$\boldsymbol{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$$

[2 marks]

b(i) and (ii). (i) attempt to substitute $t = 2$ into the equation ***(M1)*** e.g. $\begin{pmatrix} 2 \\ 6 \\ -4 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$

$\overrightarrow{OP} = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$ ***A1 N2***

(ii) correct substitution into formula for magnitude ***A1***

e.g. $\sqrt{3^2 + 5^2 + -2^2}$, $\sqrt{3^2 + 5^2 + 2^2}$

$|\overrightarrow{OP}| = \sqrt{38}$ ***A1 N1***

[4 marks]

Examiners report

a. Many candidates answered this question well. Some continue to write the vector equation in (a) using "$L =$", which does not earn full marks.

b(i) and (ii) proved accessible for most, although small arithmetic errors were not uncommon. Some candidates substituted $t = 2$ into the original equation, and a few answered $\overrightarrow{OP} = \begin{pmatrix} 2 \\ 6 \\ -4 \end{pmatrix}$. A small but surprising number of candidates left this question blank, suggesting the topic was not given adequate attention in course preparation.

Let L_x be a family of lines with equation given by $r = \begin{pmatrix} x \\ \frac{2}{x} \end{pmatrix} + t\begin{pmatrix} x^2 \\ -2 \end{pmatrix}$, where $x > 0$.

a. Write down the equation of L_1. [2]

b. A line L_a crosses the y-axis at a point P. [6]

Show that P has coordinates $\left(0, \frac{4}{a}\right)$.

c. The line L_a crosses the x-axis at $\mathrm{Q}(2a, 0)$. Let $d = \mathrm{PQ}^2$. [2]

Show that $d = 4a^2 + \frac{16}{a^2}$.

d. There is a minimum value for d. Find the value of a that gives this minimum value. [7]

Markscheme

a. attempt to substitute $x = 1$ ***(M1)***

eg $\boldsymbol{r} = \begin{pmatrix} 1 \\ \frac{2}{1} \end{pmatrix} + t\begin{pmatrix} 1^2 \\ -2 \end{pmatrix}$, $L_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

correct equation (vector or Cartesian, but do not accept "L_1")

eg $\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, $y = -2x + 4$ (must be an equation) ***A1*** ***N2***

[2 marks]

b. appropriate approach ***(M1)***

eg $\begin{pmatrix} 0 \\ y \end{pmatrix} = \begin{pmatrix} a \\ \frac{2}{a} \end{pmatrix} + t\begin{pmatrix} a^2 \\ -2 \end{pmatrix}$

correct equation for x-coordinate ***A1***

eg $0 = a + ta^2$

$t = \frac{-1}{a}$ ***A1***

substituting **their** parameter to find y ***(M1)***

eg $y = \frac{2}{a} - 2\left(\frac{-1}{a}\right)$, $\begin{pmatrix} a \\ \frac{2}{a} \end{pmatrix} - \frac{1}{a}\begin{pmatrix} a^2 \\ -2 \end{pmatrix}$

correct working ***A1***

eg $y = \frac{2}{a} + \frac{2}{a}$, $\begin{pmatrix} a \\ \frac{2}{a} \end{pmatrix} - \begin{pmatrix} a \\ -\frac{2}{a} \end{pmatrix}$

finding correct expression for y ***A1***

eg $y = \frac{4}{a}$, $\begin{pmatrix} 0 \\ \frac{4}{a} \end{pmatrix}$ $\mathrm{P}\left(0,\ \frac{4}{a}\right)$ ***AG*** ***N0***

[6 marks]

c. valid approach ***M1***

eg distance formula, Pythagorean Theorem, $\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} 2a \\ -\frac{4}{a} \end{pmatrix}$

correct simplification ***A1***

eg $(2a)^2 + \left(\frac{4}{a}\right)^2$

$d = 4a^2 + \frac{16}{a^2}$ ***AG*** ***N0***

[2 marks]

d. recognizing need to find derivative ***(M1)***

eg $d',\ d'(a)$

correct derivative ***A2***

eg $8a - \frac{32}{a^3}$, $8x - \frac{32}{x^3}$

setting **their** derivative equal to 0 ***(M1)***

eg $8a - \frac{32}{a^3} = 0$

correct working ***(A1)***

eg $8a = \frac{32}{a^3}$, $8a^4 - 32 = 0$

working towards solution ***(A1)***

eg $a^4 = 4,\ a^2 = 2,\ a = \pm\sqrt{2}$

$a = \sqrt[4]{4}$ $(a = \sqrt{2})$ (do not accept $\pm\sqrt{2}$) ***A1*** ***N3***

[7 marks]

Total [17 marks]

Examiners report

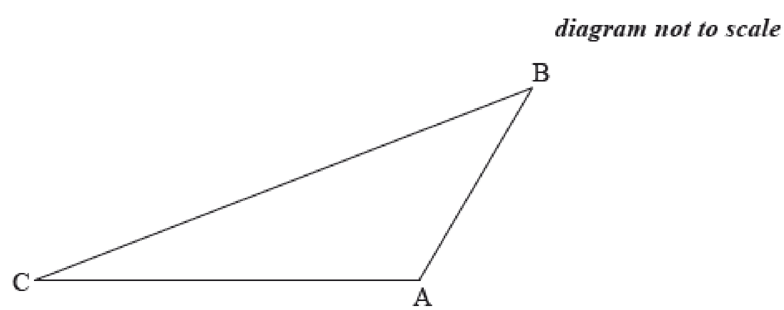
a. In part (a), most candidates correctly substituted 1 for x, although many of them did not earn full marks for their work here, as they wrote their vector equation using $L_1 =$, not understanding that L_1 is the name of the line, and not a vector.

b. Very few candidates answered parts (b) and (c) correctly, often working backwards from the given answer, which is not appropriate in "show that" questions. In these types of questions, candidates are required to clearly show their working and reasoning, which will hopefully lead them to the given answer.

c. Very few candidates answered parts (b) and (c) correctly, often working backwards from the given answer, which is not appropriate in "show that" questions. In these types of questions, candidates are required to clearly show their working and reasoning, which will hopefully lead them to the given answer.

d. Fortunately, a good number of candidates recognized the need to find the derivative of the given expression for d in part (d) of the question, and so were able to earn at least some of the available marks in the final part.

The following diagram shows triangle ABC.



Let $\overrightarrow{\mathrm{AB}} \bullet \overrightarrow{\mathrm{AC}} = -5\sqrt{3}$ and $\left|\overrightarrow{\mathrm{AB}}\right|\left|\overrightarrow{\mathrm{AC}}\right| = 10$. Find the area of triangle ABC.

Markscheme

attempt to find $\cos \mathrm{C\hat{A}B}$ (seen anywhere) ***(M1)***

eg $\cos\theta = \frac{\overrightarrow{\mathrm{AB}}\bullet\overrightarrow{\mathrm{AC}}}{\left|\overrightarrow{\mathrm{AB}}\right|\left|\overrightarrow{\mathrm{AC}}\right|}$

$\cos \mathrm{C\hat{A}B} = \frac{-5\sqrt{3}}{10} \quad \left(= -\frac{\sqrt{3}}{2}\right)$ ***A1***

valid attempt to find $\sin \mathrm{C\hat{A}B}$ ***(M1)***

eg triangle, Pythagorean identity, $\mathrm{C\hat{A}B} = \frac{5\pi}{6},\ 150^\circ$

$\sin \mathrm{C\hat{A}B} = \frac{1}{2}$ ***(A1)***

correct substitution into formula for area ***(A1)*** *eg* $\frac{1}{2} \times 10 \times \frac{1}{2}$, $\frac{1}{2} \times 10 \times \sin \frac{\pi}{6}$

area $= \frac{10}{4}$ $\left(= \frac{5}{2}\right)$ ***A1*** ***N3***

[6 marks]

Examiners report

The large majority of candidates were able to find the correct expression for $\cos \mathrm{C\hat{A}B}$, but few recognized that an angle with a negative cosine will be obtuse, rather than acute, and many stated that $\mathrm{C\hat{A}B} = 30^{\circ}$. When substituting into the triangle area formula, a common error was to substitute $5\sqrt{3}$ rather than 10, as many did not understand the relationship between the magnitude of a vector and the length of a line segment in the triangle formula.

Some of the G2 comments from schools suggested that it might have been easier for their students if this question were split into two parts. While we do tend to provide more support on the earlier questions in the paper, questions 6 and 7 are usually presented with little or no scaffolding. On these later questions, the candidates are often required to use knowledge from different areas of the syllabus within a single question.

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a. Let $\boldsymbol{u} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 3 \\ -1 \\ p \end{pmatrix}$. Given that $\boldsymbol{u}$ is perpendicular to $\boldsymbol{w}$, find the value of p. [3]

b. Let $\boldsymbol{v} = \begin{pmatrix} 1 \\ q \\ 5 \end{pmatrix}$. Given that $|\boldsymbol{v}| = \sqrt{42}$, find the possible values of q. [3]

Markscheme

a. evidence of equating scalar product to 0 ***(M1)***

$2 \times 3 + 3 \times (-1) + (-1) \times p = 0 \quad (6 - 3 - p = 0, 3 - p = 0)$ ***A1***

$p = 3$ ***A1*** ***N2***

[3 marks]

b. evidence of substituting into magnitude formula ***(M1)***

e.g. $\sqrt{1 + q^2 + 25}$, $1 + q^2 + 25$

setting up a correct equation ***A1***

e.g. $\sqrt{1 + q^2 + 25} = \sqrt{42}$, $1 + q^2 + 25 = 42$, $q^2 = 16$

$q = \pm 4$ ***A1*** ***N2***

[3 marks]

Examiners report

a. Most candidates knew to set the scalar product equal to zero.

b. Most candidates knew to set the scalar product equal to zero. A pleasing number found both answers for q, although some often neglected to provide both solutions.

Find the cosine of the angle between the two vectors $3\boldsymbol{i} + 4\boldsymbol{j} + 5\boldsymbol{k}$ and $4\boldsymbol{i} - 5\boldsymbol{j} - 3\boldsymbol{k}$.

Markscheme

finding scalar product and magnitudes ***(A1)(A1)(A1)***

scalar product $= 12 - 20 - 15\ (= -23)$

magnitudes $= \sqrt{3^2 + 4^2 + 5^2}\ , = \sqrt{4^2 + (-5)^2 + (-3)^2}\ , \left(\sqrt{50}, \sqrt{50}\right)$

substitution into formula ***M1***

e.g. $\cos\theta = \frac{12-20-15}{\left(\sqrt{3^2+4^2+5^2}\right)\times\left(\sqrt{4^2+(-5)^2+(-3)^2}\right)}$

$\cos\theta = -\frac{23}{50}\ (= -0.46)$ ***A2 N4***

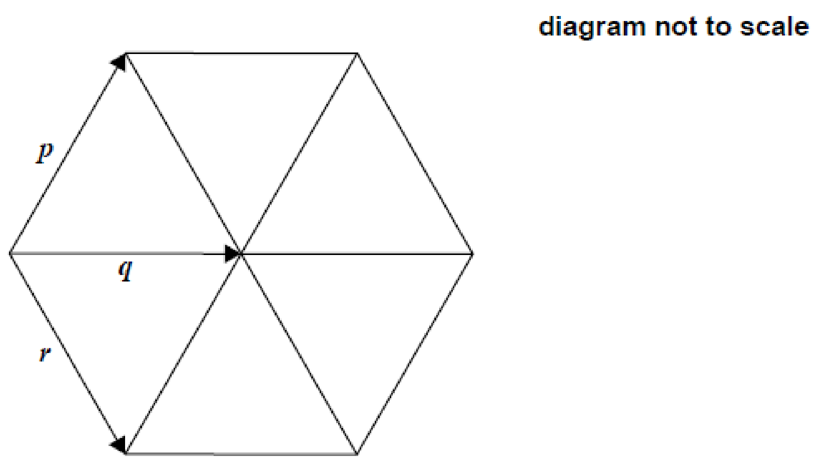
[6 marks]

Examiners report

Many candidates performed well in finding the magnitudes and scalar product to use the formula for angle between vectors. Some experienced trouble with the arithmetic to obtain the required result. A significant number of candidates isolated the theta answering with $\arccos\left(\frac{-23}{50}\right)$.

Six equilateral triangles, each with side length 3 cm, are arranged to form a hexagon.

This is shown in the following diagram.



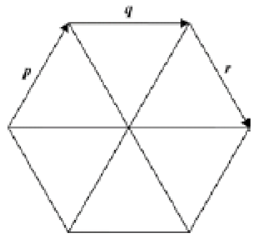
The vectors ***p*** , ***q*** and ***r*** are shown on the diagram.

Find ***p***•(***p*** + ***q*** + ***r***).

Markscheme

METHOD 1 (using |***p***| |2***q***| cosθ)

finding ***p*** + ***q*** + ***r*** ***(A1)***

eg 2***q***, 

| ***p*** + ***q*** + ***r*** | = 2 × 3 (= 6) (seen anywhere) ***A1***

correct angle between ***p*** and ***q*** (seen anywhere) ***(A1)***

$\frac{\pi}{3}$ (accept 60°)

substitution of **their** values ***(M1)***

eg 3 × 6 × cos$\left(\frac{\pi}{3}\right)$

correct value for cos$\left(\frac{\pi}{3}\right)$ (seen anywhere) ***(A1)***

eg $\frac{1}{2}$, $3 \times 6 \times \frac{1}{2}$

p•(***p*** + ***q*** + ***r***) = 9 ***A1 N3***

METHOD 2 (scalar product using distributive law)

correct expression for scalar distribution ***(A1)***

eg ***p***• ***p*** + ***p***•***q*** + ***p***•***r***

three correct angles between the vector pairs (seen anywhere) ***(A2)***

eg 0° between ***p*** and ***p***, $\frac{\pi}{3}$ between ***p*** and ***q***, $\frac{2\pi}{3}$ between ***p*** and ***r***

Note: Award ***A1*** for only two correct angles.

substitution of **their** values ***(M1)***

eg 3.3.cos0 +3.3.cos$\frac{\pi}{3}$ + 3.3.cos120

one correct value for cos0, cos$\left(\frac{\pi}{3}\right)$ or cos$\left(\frac{2\pi}{3}\right)$ (seen anywhere) ***A1***

eg $\frac{1}{2}$, $3 \times 6 \times \frac{1}{2}$

p•(***p*** + ***q*** + ***r***) = 9 ***A1 N3***

METHOD 3 (scalar product using relative position vectors)

valid attempt to find one component of ***p*** or ***r*** ***(M1)***

eg sin 60 = $\frac{x}{3}$, cos 60 = $\frac{x}{3}$, one correct value $\frac{3}{2}$, $\frac{3\sqrt{3}}{2}$, $\frac{-3\sqrt{3}}{2}$

one correct vector (two or three dimensions) (seen anywhere) ***A1***

eg $p = \begin{pmatrix} \frac{3}{2} \\ \frac{3\sqrt{3}}{2} \end{pmatrix}$, $q = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$, $r = \begin{pmatrix} \frac{3}{2} \\ -\frac{3\sqrt{3}}{2} \\ 0 \end{pmatrix}$

three correct vectors ***p*** + ***q*** + ***r*** = 2***q*** ***(A1)***

$\boldsymbol{p} + \boldsymbol{q} + \boldsymbol{r} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$ (seen anywhere, including scalar product) ***(A1)***

correct working ***(A1)***

eg $\left(\frac{3}{2} \times 6\right) + \left(\frac{3\sqrt{3}}{2} \times 0\right),\ 9 + 0 + 0$

$\boldsymbol{p} \cdot (\boldsymbol{p} + \boldsymbol{q} + \boldsymbol{r}) = 9$ ***A1 N3***

[6 marks]

Examiners report

[N/A]

In the following diagram, $\overrightarrow{\mathrm{OP}} = \boldsymbol{p}$, $\overrightarrow{\mathrm{OQ}} = \boldsymbol{q}$ and $\overrightarrow{\mathrm{PT}} = \frac{1}{2}\overrightarrow{\mathrm{PQ}}$.

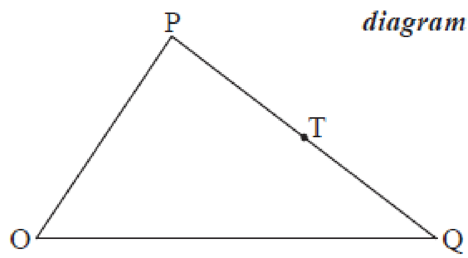


diagram not to scale

Express each of the following vectors in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$,

a. $\overrightarrow{\mathrm{QP}}$; [2]

b. $\overrightarrow{\mathrm{OT}}$. [3]

Markscheme

a. appropriate approach ***(M1)***

eg $\overrightarrow{\mathrm{QP}} = \overrightarrow{\mathrm{QO}} + \overrightarrow{\mathrm{OP}},\ \mathrm{P} - \mathrm{Q}$

$\overrightarrow{\mathrm{QP}} = \boldsymbol{p} - \boldsymbol{q}$ ***A1 N2***

[2 marks]

b. recognizing correct vector for $\overrightarrow{\mathrm{QT}}$ or $\overrightarrow{\mathrm{PT}}$ ***(A1)***

eg $\overrightarrow{\mathrm{QT}} = \frac{1}{2}(\boldsymbol{p} - \boldsymbol{q}), \overrightarrow{\mathrm{PT}} = \frac{1}{2}(\boldsymbol{q} - \boldsymbol{p})$

appropriate approach ***(M1)***

eg $\overrightarrow{\mathrm{OT}} = \overrightarrow{\mathrm{OP}} + \overrightarrow{\mathrm{PT}},\ \overrightarrow{\mathrm{OQ}} + \overrightarrow{\mathrm{QT}},\ \overrightarrow{\mathrm{OP}} + \frac{1}{2}\overrightarrow{\mathrm{PQ}}$

$\overrightarrow{\mathrm{OT}} = \frac{1}{2}(\boldsymbol{p} + \boldsymbol{q})$ $\left(\text{accept } \frac{p+q}{2}\right)$ ***A1 N2***

[3 marks]

Examiners report

a. [N/A]
[N/A]

b.

Point A has coordinates (−4, −12, 1) and point B has coordinates (2, −4, −4).

The line *L* passes through A and B.

a. Show that $\vec{\mathrm{AB}} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$ [1]

b.i. Find a vector equation for *L*. [2]

b.ii. Point *C* (*k* , 12 , −*k*) is on *L*. Show that *k* = 14. [4]

c.i. Find $\vec{\mathrm{OB}} \bullet \vec{\mathrm{AB}}$. [2]

c.ii. Write down the value of angle OBA. [1]

d. Point D is also on *L* and has coordinates (8, 4, −9). [6]

Find the area of triangle OCD.

Markscheme

a. correct approach ***A1***

eg $\vec{\mathrm{AO}} + \vec{\mathrm{OB}},\ \mathrm{B} - \mathrm{A},\ \begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix} - \begin{pmatrix} -4 \\ -12 \\ 1 \end{pmatrix}$

$\vec{\mathrm{AB}} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$ ***AG N0***

[1 mark]

b.i. any correct equation in the form ***r*** = ***a*** + *t**b*** (any parameter for *t*) ***A2 N2***

where ***a*** is $\begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix}$ or $\begin{pmatrix} -4 \\ -12 \\ 1 \end{pmatrix}$ and ***b*** is a scalar multiple of $\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$

eg $\boldsymbol{r} = \begin{pmatrix} -4 \\ -12 \\ 1 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix},\ (x,\ y,\ z) = (2,\ -4,\ -4) + t\,(6,\ 8,\ -5)\,, \boldsymbol{r} = \begin{pmatrix} -4+6t \\ -12+8t \\ 1-5t \end{pmatrix}$

Note: Award ***A1*** for the form ***a*** + *t**b***, ***A1*** for the form ***L*** = ***a*** + *t**b***, ***A0*** for the form ***r*** = ***b*** + *t**a***.

[2 marks]

b.ii. **METHOD 1** (solving for *t*)

valid approach ***(M1)***

eg $\begin{pmatrix} k \\ 12 \\ -k \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix},\ \begin{pmatrix} k \\ 12 \\ -k \end{pmatrix} = \begin{pmatrix} -4 \\ -12 \\ 1 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$

one correct equation ***A1***

eg $-4 + 8t = 12$, $-12 + 8t = 12$

correct value for t ***(A1)***

eg $t = 2$ or 3

correct substitution ***A1***

eg $2 + 6(2)$, $-4 + 6(3)$, $-[1 + 3(-5)]$

$k = 14$ ***AG N0***

METHOD 2 (solving simultaneously)

valid approach ***(M1)***

eg $\begin{pmatrix} k \\ 12 \\ -k \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}, \begin{pmatrix} k \\ 12 \\ -k \end{pmatrix} = \begin{pmatrix} -4 \\ -12 \\ 1 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$

two correct equations in ***A1***

eg $k = -4 + 6t$, $-k = 1 - 5t$

EITHER (eliminating k)

correct value for t ***(A1)***

eg $t = 2$ or 3

correct substitution ***A1***

eg $2 + 6(2)$, $-4 + 6(3)$

OR (eliminating t)

correct equation(s) ***(A1)***

eg $5k + 20 = 30t$ **and** $-6k - 6 = 30t$, $-k = 1 - 5\left(\frac{k+4}{6}\right)$

correct working clearly leading to $k = 14$ ***A1***

eg $-k + 14 = 0$, $-6k = 6 - 5k - 20$, $5k = -20 + 6(1 + k)$

THEN

$k = 14$ ***AG N0***

[4 marks]

c.i. correct substitution into scalar product ***A1***

eg $(2)(6) - (4)(8) - (4)(-5)$, $12 - 32 + 20$

$\vec{\mathrm{OB}} \bullet \vec{\mathrm{AB}} = 0$ ***A1 N0***

[2 marks]

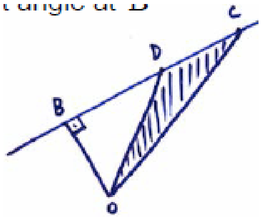
c.ii. $\mathrm{O}\hat{\mathrm{B}}\mathrm{A} = \frac{\pi}{2},\ 90^\circ \quad \left(\text{accept } \frac{3\pi}{2},\ 270^\circ\right)$ ***A1 N1***

[1 marks]

d. **METHOD 1** ($\frac{1}{2}$ × height × CD)

recognizing that OB is altitude of triangle with base CD (seen anywhere) ***M1***

eg $\frac{1}{2} \times \left|\vec{\mathrm{OB}}\right| \times \left|\vec{\mathrm{CD}}\right|$, $\mathrm{OB} \perp \mathrm{CD}$, sketch showing right angle at B t angle at B



$\vec{CD} = \begin{pmatrix} -6 \\ -8 \\ 5 \end{pmatrix}$ or $\vec{DC} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$ (seen anywhere) ***(A1)***

correct magnitudes (seen anywhere) ***(A1)(A1)***

$\left|\vec{OB}\right| = \sqrt{(2)^2 + (-4)^2 + (-4)^2} = \left(\sqrt{36}\right)$

$\left|\vec{CD}\right| = \sqrt{(-6)^2 + (-8)^2 + (5)^2} = \left(\sqrt{125}\right)$

correct substitution into $\frac{1}{2}bh$ ***A1***

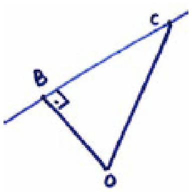
eg $\frac{1}{2} \times 6 \times \sqrt{125}$

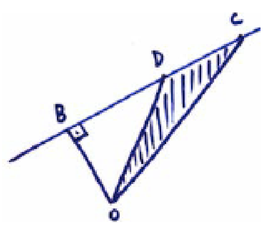
area $= 3\sqrt{125},\ 15\sqrt{5}$ ***A1 N3***

METHOD 2 (subtracting triangles)

recognizing that OB is altitude of either ΔOBD or ΔOBC(seen anywhere) ***M1***

eg $\frac{1}{2} \times \left|\vec{OB}\right| \times \left|\vec{BD}\right|,\ \mathrm{OB} \perp \mathrm{BC}$, sketch of triangle showing right angle at B





one correct vector $\vec{BD}$ or $\vec{DB}$ or $\vec{BC}$ or $\vec{CB}$ (seen anywhere) ***(A1)***

eg $\vec{BD} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}, \vec{CB} = \begin{pmatrix} -12 \\ -16 \\ 10 \end{pmatrix}$

$\left|\vec{OB}\right| = \sqrt{(2)^2 + (-4)^2 + (-4)^2} = \left(\sqrt{36}\right)$ (seen anywhere) ***(A1)***

one correct magnitude of a base (seen anywhere) ***(A1)***

$\left|\vec{BD}\right| = \sqrt{(6)^2 + (8)^2 + (5)^2} = \left(\sqrt{125}\right),\ \left|\vec{BC}\right| = \sqrt{144 + 256 + 100} = \left(\sqrt{500}\right)$

correct working ***A1***

eg $\frac{1}{2} \times 6 \times \sqrt{500} - \frac{1}{2} \times 6 \times 5\sqrt{5},\ \frac{1}{2} \times 6 \times \sqrt{500} \times \sin 90 - \frac{1}{2} \times 6 \times 5\sqrt{5} \times \sin 90$

area $= 3\sqrt{125},\ 15\sqrt{5}$ ***A1 N3***

METHOD 3 (using $\frac{1}{2}ab \sin C$ with ΔOCD)

two correct side lengths (seen anywhere) ***(A1)(A1)***

$\left|\vec{OD}\right| = \sqrt{(8)^2 + (4)^2 + (-9)^2} = \left(\sqrt{161}\right),\ \left|\vec{CD}\right| = \sqrt{(-6)^2 + (-8)^2 + (5)^2} = \left(\sqrt{125}\right),$
$\left|\vec{OC}\right| = \sqrt{(14)^2 + (12)^2 + (-14)^2} = \left(\sqrt{536}\right)$

attempt to find cosine ratio (seen anywhere) ***M1***

eg $\frac{536-286}{-2\sqrt{161}\sqrt{125}}$, $\frac{\text{OD}\bullet\text{DC}}{|OD||DC|}$

correct working for sine ratio ***A1***

eg $\frac{(125)^2}{161\times125}+\sin^2 D=1$

correct substitution into $\frac{1}{2}ab\,\sin C$ ***A1***

eg $0.5\times\sqrt{161}\times\sqrt{125}\times\frac{6}{\sqrt{161}}$

area $=3\sqrt{125},\ 15\sqrt{5}$ ***A1 N3***

[6 marks]

Examiners report

a. [N/A]
b.i. [N/A]
b.ii. [N/A]
c.i. [N/A]
c.ii. [N/A]
d. [N/A]